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Money supply determination and a lagged reserve accounting system

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Money supply determination and a lagged reserve accounting system

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DEDICATION

To My Faithful Wife

تقدیم به همسر عزیز و با ایمانم "آذر" که با صبری غیر قابل وصف در جمع آوری
این مجموعه مرا یاری کرد

PREFACE

بسم الله الرحمن الرحیم

In the Name of God, Most Gracious, Most Merciful

در کتاب انسان، حکایتی از کویریان رفته است که گویای ابعاد متعالی سیر از طریقت انسانی است. اشارتی به سفرهای طولانی دیگر زمین و به معراج روحانی در آسمان این کویر که درد عشق را بیان می‌کند، از پروازها و فرودها، از طلوع‌ها و غروب‌ها، از نشیب‌ها و فرازها و از دیگر اسرار رمزها سخن رفته است که داعی بر ادراکی است که انسان در کتاب خود اندیشه کند و خویش را دریابد که خدایش را دریافته باشد (من عرف نفسه فقد عرف ربه).

ترسیم اسلاف و اخلاف زمین و زمان و تبیین حدیث فراگیرنده (اغانشید) در آن همه بر آنند که توفیق، رفیق عباد صالح است و این حقیقت مطلق در چهارچوب حکمت متجلی است. و امروز که دو سال از تحقق بزرگترین آرزوی مستضعفان جهان عموماً و ایران مستضعف خصوصاً می‌گذرد، جای حمد است که خداوند نعمت عظمای هدایت مسلمان ایران به قیادت رهبر کبیر امام خمینی عنایت فرموده. آن وعده خود حضرت حق است که باید (کتب الله لاغلبن انا و رسلی) مصداق بگیرد.

من که در دوران انقلاب در کنار خواهران و برادران خود بودم و مشغول تهیه این مجموعه که نتیجه تلاش پنج سال است بودم، رجاء دارم که این اندوخته را در راه اسلام

وسیله‌ای استخدام کنم هرچند اندک باشد. امید که چنین باد.

والسلام

بیست و دوم بهمن هزار و سیصد و پنجاه و چند هجری شمسی

سعدآباد - پرویز داودی

CHAPTER I. INTRODUCTION AND GENERAL DISCUSSION

Basic Problem

Legal commercial-bank reserve requirements have an important influence upon the size of the money stock and the ability of the central bank to control it.

Most of the commercial banks in the United States must satisfy legal reserve requirements. For state chartered banks which are not members of the Federal Reserve System, reserve requirements are set by agencies of the state governments. Commercial banks which are members of the Federal Reserve System (regardless of whether they are chartered by the federal government or by state governments) must satisfy legal reserve requirements which are set by the Board of Governors of the Federal Reserve System. The Board's rules concerning member bank reserve requirements are known as "Regulation D."

In September, 1968, the Board radically changed Regulation D. Before that date, the Federal Reserve's reserve-requirement system involved "contemporaneous reserve accounting." After September, 1968, "lagged reserve accounting" was in effect.

Under contemporaneous reserve accounting (CRA), the average reserves which a bank must hold during any particular week are a function of its average deposit liabilities during that same week. Under lagged reserve accounting (LRA), the average reserves which a bank must hold during the week are a function of its average deposit liabilities during the week before last. Furthermore, under CRA, "legal" reserves consisted of current holdings of vault cash (i.e., currency and coin) and current deposit balances with a Federal Reserve Bank. Under LRA, legal reserves consist of current deposit balances with a Federal Reserve Bank and vault cash held two weeks earlier. Hence, current holdings of vault cash do not meet the criteria of "legal reserves" under LRA.

The practical problems which led the Board of Governors of the Federal Reserve System to vote for the 1968 revision can be explained with a description of member banks which were operating under CRA.

Under CRA, the reserve settlement week ended on Wednesday morning. Therefore, on Tuesday, member banks were at the peak of their efforts to achieve their required reserve balances. When this could not be fulfilled by routine deposit of reserve balances, banks had to search for ways to vary their loans and investments to the nonbank-public. This was done in order to contract or expand deposit liabilities in response to shortages or surpluses of reserves from required amounts. Of course, policy actions by the Board of Governors of the Federal Reserve System and currency holdings by the nonbank-public can and will affect the deposit and reserve situation of the banks. However, these effects are not under the banks' control. Other bank transactions such as selling or buying secondary reserves, issuing or buying certificates of deposits, and other similar actions, are useless in terms of changing the amount of reserves in the banking system, as long as currency in the hands of the nonbank-public is unaffected. And moreover, there exists a general reluctance for the banks to borrow from the Federal Reserve Banks' discount windows. All of these elements reinforce the idea that the ability of the banking system to escape the shortages or surpluses of reserves on Wednesdays, rested on its ability to contract or expand its deposit liabilities by changing loans and investments to the nonbank-public. The problem was obvious. Tuesdays of every week turned out to be days with (1) high fluctuations in interest rates, (2) intensive defensive operations by the Board of Governors of the Federal Reserve System for the purpose of stabilizing its monetary policy target variables, and (3) great uncertainty for banks.

The purpose of the 1968 revision was to decrease the amount of uncertainty which banks were encountering, by giving them enough time to adjust their required reserves. The expectation was that the fluctuation of interest rates would be smoothed out and the transaction of the reserves would be less concentrated on one single day.

Since 1968, there have been many studies that have criticized the new system on both theoretical and empirical grounds. Coats (12), Burger (8), Pierce (31), Gilbert (19) and Laurent (27) can be cited.

The main theoretical weakness of the new system, agreed upon by most of the studies, lies in the absence of a link between current required reserves and current deposits. This problem causes destruction of the feedback mechanism of changes going from the current reserves situation due to the policy action of the Federal Reserve (i.e., change in total or unborrowed reserves) to current required reserves via changes in current deposits. Hence, no limit or level of consequent change in deposits can be estimated.

The other criticism of the system concerns the banks' behavior. The absence of immediate worry for their required reserves under this system would encourage banks to have very large amounts of deposit liabilities due to large loans and investments made to the nonbank-public. The problem, then, is that after two weeks, the banks must worry about acquiring a large amount of required reserves, due to large accumulations of deposits from the previous two weeks. Banks cannot travel backwards in time to decrease those deposits. Individual banks, then, intensively desire new reserves, which are difficult to acquire for the banking system as a whole. Eventually, for the purpose of preventing reserve deficiency, Federal Reserve Banks will let banks use their discount windows, which causes the Federal Reserve authorities to depart from their target and pursue a policy which may not follow their target path. Moreover, intensive demand for reserves, which has been created by this problem, could push interest rates higher. This might be another reason for the more active Federal Reserve defensive policy action under LRA.

As a result, the banks might follow a very conservative policy in acquiring deposits for the following two weeks. In turn, it would create a problem in reverse, which would result after four weeks in a reserve surplus, intensive downward push of interest rates, and again an intensive Federal Reserve defensive operation. Therefore, adoption of this new system would introduce a two week period of fluctuations in interest rates and money market variables, less control on deposit creation, and more problems in pursuing the monetary policies by the Federal Reserve.

Empirical research results support the intensity of the mentioned problems under LRA; see Coats (12) and Burger (8).

Objectives of the Study

The objectives of this study are twofold. The first is to build a model which includes the system of LRA in the mechanism by which Federal Reserve actions are transmitted to financial markets based on different behavioral assumptions for both the banking system and the Federal Reserve System. In the process of developing the analysis, it will be shown how heavily the banking system relies on its past behavior as a consequence of the inclusion of lagged accounting.

Secondly, under a certain set of behavioral, economical and statistical assumptions, the impact of the Federal Reserve Banks' policy action on the banking deposit will be examined and estimated. The key variables, such as interest rates and bank deposits, will be forecasted.

Outline of the Study

In Chapter II, some of the related literature on the system of lagged reserves accounting and short-term banking behavior will be reviewed.

Chapter III consists of four parts. Abbreviations of commonly used variables are presented in part one. In part two, a summary of the study by Modigliani, Rasche and Cooper (30) is presented. The system of lagged reserve accounting is included in the model, and the deposit supply function is derived. In part three, the model is modified by adding certificates of deposits and Eurodollar borrowings to the banking behavioral functions. The modified version of deposit supply function is developed. In part four, "Federal fund rate" is used as an instrumental variable. Repurchase agreement activities are considered in banks' behavioral adjustments, and new supply relations are derived.

In Chapter IV, statistical problems such as seasonal variations and serial correlations are described in part one. Parts two, three and four present the results of fitting deposit supply functions with ordinary-least-square and with the simultaneous equations technique of two-stage-least-squares. In Part V, the ability of Model I under LRA to generate the historical values of interest rate and deposit supply, for the period of July, 1975, to June, 1976, is tested.

Chapter V consists of summary and conclusions.

CHAPTER II. A REVIEW OF RELATED LITERATURE

There are a few studies on the subject of lagged reserves accounting. The practical problems represented in Chapter One are agreed upon by most of the studies. For example, Laurent (27) concludes that the problems of high management cost for individual banks, more viability of interest rates and deposits, and the difficulties of conducting monetary policy by the Federal Reserve authorities are worsened under the lagged accounting system. Gilbert (19), Laufenberg (26), Coats (12) and Pierce (31) also reach the same conclusions.

To solve the problems of the new system, Laurent (27, pp. 18-20) proposes a system in which the direction of the lag between deposits and required reserves is reversed. Under the proposed system, the Federal Reserve authorities would set the amount of reserves during the current week, and the level of required reserves would be determined by the deposit creation of the banks two weeks later.

Pierce (31) suggests the use of the marginal reserves as opposed to the total reserves as policy control variables. In addition to that, he substitutes what he calls "required contemporaneous reserves" for required reserves under the lagged accounting system. "Required contemporaneous reserves" could be satisfied with the deposited reserves of the banks at the Federal Reserve Banks at the current week plus the vault cash of the banks in the current week.

All of the studies mentioned above have some kind of implicit behavioral assumptions for both the individual banks and the banking system. Among these studies, Coats (12) and Laufenberg (26) explicitly devise the behavioral assumptions for the banking system, but still the individual banks' behavioral assumptions are implicit in their studies.

Laufenberg (26) in his study formulates the behavioral assumptions for the banking system as:

1) Equilibrium condition: $FR_t^d = FR_t$

2) $FR_t^d = F(i_t, i_{d_t}) + \gamma D_t$

where

FR_t = Free reserves in the banking system;

FR_t^d = Desired level of free reserves in the banking system;

i_t = Federal funds interest rate;

i_{d_t} = Federal Reserve discount rate;

γ = Constant term;

D_t = Demand deposit of the member banks.

Furthermore, i can be considered as the opportunity cost of holding excess reserves and alternative cost to the cost of borrowing from the Federal Reserve Banks (FRB). Therefore, an increase (decrease) in i would induce the banks to hold less (more) excess reserves and, hence, less (more) free reserves. Which means: $\frac{\partial FR^d}{\partial i} < 0$. On the other hand, the higher (lower) the FRB's discount rate "ceteris paribus," the smaller (larger) the bank's desire to borrow from the FRB. Therefore, the larger (smaller) free reserve is desired by the banks. Or: $\frac{\partial FR^d}{\partial d} > 0$. It is also assumed that: $\frac{\partial FR^d}{\partial D} = \gamma > 0$. The higher (lower) the deposit level will induce banks desire to hold more (less) free reserves.

Whenever there is a gap between the actual and the desired levels of the free reserves, there will be incentive for the banks to close the gap. Specifically, if the banks' actual free reserves increase due to policy action by the Federal Reserve authorities, the banks will be encouraged to make more loans and investments to the nonbank-public and to create more deposits. Under contemporaneous reserve accounting, as a result of the above actions, the banks' required reserves will increase and absorb the reserves from the actual free reserves. Hence, the level of actual free reserves will decrease and close the disequilibrium gap.

Under the lagged reserve accounting, the larger deposits of the banks will not affect the current level of the required reserves, so there will be no mechanism by which the required reserves absorb the extra free reserves created by the policy actions.

In his second behavioral assumption, for the purpose of solving this problem, Laufenberg assumes the desired level of free reserves will get larger as the amounts of the deposits increase. The banks' reactions will restore the equilibrium, not by decreasing the actual level of the free reserves, but by increasing their desired level. Variations in both rates i and i_d help to close the disequilibrium gap faster by affecting the desired level of the free reserves, and the burden on these rates for closing the disequilibrium gap is higher under the lagged accounting system.

Based on his assumption, Laufenberg derives deposit supply functions under both the lagged and the contemporaneous accounting systems. He discusses the effects of changes in the policy variables under both systems. He found stable, continuous and dynamic paths for the interest rate and the level of the deposit to their new equilibriums as a result of the adjustment for the changes in the policy variables under the contemporaneous accounting system. Further, he found an oscillating dynamic path for the variables under lagged reserve accounting. His analysis can be extended to a case in which the stability of the system depends upon certain conditions imposed on the elasticities of the supply and demand curves for deposits.

As mentioned before, in all of these studies, a complete and explicit set of behavioral assumptions at the micro-level (the individual bank), and at the macro-level (the banking system) is absent.

The literature in short-run banking behavior can be classified into two parts. The first approach imposes behavioral equations based on a combination of environmental factors affecting banks. The applications of the familiar stock-adjustment models are heavily used in this approach. The studies by Fraser and Rose (16), Bryan and Carleton (6) and the studies by Coats (12) and Laufenberg (26) (even though they do not talk about the stock-adjustment models) can be mentioned.

For example, Bryan and Carleton (6), as their first hypothesis for the individual banks' short-term behavior, impose the following relationships:

$$R^E_{t+1} - R^E_t = f\left[\left(R^{E*}_{t+1} - R^E_t\right), \left(D_{t+1} - D_t\right), E_t\right] \qquad (1)$$

$$R^{E*}_{t+1} = h\left[r_t, B_t, L_t, M_t, S_t\right] \qquad (2)$$

where:

R^E = All of the nonearning assets - required reserves;

R^{E*} = The desired level of R^E;

D = Demand deposits;

r = A measure for either Treasury bill rate, or marginal rate on investment, or Federal fund interest rate;

B = A measure for either outstanding borrowings, or cost of borrowings, or borrowed reserves;

L = Loan demand;

S = Bank's size;

E_t = The disturbance term;

M = Deposit mix = $\dfrac{\text{savings + time deposits}}{\text{total deposits}}$.

Then they test the hypothesis with different combinations of variables and measures for r and B. They use a similar approach for their second hypothesis.

In the second approach, the analytical apparatus and structure of the system develops in a framework which is based on well-known behavioral assumptions such as profit maximization, utility maximization, or cost minimization, with or without some kind of constraints.

Studies by Modigliani, Rasche and Cooper (30), Hester and Pierce (22) and Wood (38, pp. 1-63) assume banks are profit maximizers. Aigner (1), and Aigner and Bryan (2), use the utility maximizing criterion for the banks. Boltersperger and Hellmuth (4) assume banks are cost minimizers, and Depamphilis (13) uses liquidity constrained cost minimizing criterion for the banks.

Among the above studies, Modigliani, Rasche and Cooper (30) describe the banking behavior in both the individual bank and the banking system scale, in its broadest and best form. The analyses presented in the following chapters will heavily concentrate on the model by Modigliani, Rasche and Cooper (30).

In the subject of the relationships between the Federal Reserves System and the banking system, most studies have assumed some kind of aggregate reserves as the control or instrumental variables which are used by the Federal Reserves authorities.

During the period from 1970 to 1976, which is the period used for the empirical estimations in Chapter IV of this study, the Federal Reserve authorities used the Federal fund interest rate as their instrumental variable for the purpose of achieving the desired path of the money supply. The study by Kopecky (25) takes this fact into account. Model 3 in Chapter III will discuss this point.

CHAPTER III. THEORETICAL MODELS OF MONEY SUPPLY DETERMINATION UNDER LAGGED RESERVE ACCOUNTING

Part I. Abbreviations

CRA = Contemporaneous Reserve Accounting

LRA = Lagged Reserve Accounting

FRB = Federal Reserve Banks

FFM = Federal Fund Market

FFR = Federal Fund Market Interest Rate

FOC = First Order Condition

SOC = Second Order Condition

GNP = Gross National Product

Fed = Federal Reserve Authorities

RP = Repurchase Agreement

RV = Reverse Repurchase Agreement

CD = Large-Negotiable Certificates of Deposit

OMO = Open Market Operation

FOMC = Federal Open Market Committee

USGS = The United States Government Securities

EURO = Eurodollar

FDIC = Federal Deposit Insurance Corporation

Part II. Model I

Summary of Modigliani, Rasche, and Cooper's Model (Model I under contemporaneous reserve accounting)

Assumptions

1. The supply of the currency component of money supply is demand determined so the paper concentrates on determinants of the stock of the demand deposit component of money supply.
2. Only member banks are analyzed.
3. In the short-run, the rise in commercial loan supply is demand determined. Banks accommodate the increase in the demand for commercial loans because commercial loan customers are the sources of deposits and other banking businesses.

Using the system of CRA, an individual bank's balance sheet is shown below.

Representative Bank's Balance Sheet

Assets				Liabilities		
Reserves Total			R	Demand Deposits		D
Required		RR		Federal Government	D_g	
against demand deposit	δD			All Others	D_p	
against time deposit	τT			Time Deposits		T
Surplus (excess reserve + loans in FFM)		S		Borrowing (from FRB and FFM)		B^*
Commercial Loans			CL	Miscellaneous Liabilities and Capital		$CA+MA$
Other Investments			I			
Miscellaneous Assets			MA			

____

The bank is visualized as holding anticipations as to the level of D, T, and CL which will prevail over the coming decision period. Assuming errors in anticipations, the variables D, T, and CL are randon variables subject to a known subjective probability distribution. Each variable Z is a sum of two components, a mathematical expectation $\overline{Z}$ and a random part $\tilde{X}_Z$ where Z = D,T,CL.

$$Z = \overline{Z} + X_Z. \tag{1}$$

From the balance sheet of the representative bank, the following identities are derived.

$$\begin{aligned} FR &\equiv S-B^* \\ FR &\equiv D+T-RR-CL-I+CA \end{aligned} \tag{2}$$

Under CRA: $RR = \delta D+\tau T$

δ = Required reserves ratio against D

τ = Required reserves ratio against T.

Given constant CA, for any chosen I, the outcome of FR depends upon the realization of the three random variables D, T, and CL. Hence, FR itself would be a random variable. So, from (1) and (2):

$$FR = \overline{FR}+\tilde{X}_{FR} \tag{3}$$

$$\overline{FR} = \overline{D}(1-\delta)+\overline{T}(1-\tau)-\overline{CL}-I+CA \tag{4}$$

$$\tilde{X} = \tilde{X}_D(1-\delta)+\tilde{X}_T(1-\tau)-\tilde{X}_{CL} \ . \tag{5}$$

$\phi(\tilde{X})$ is defined as the probability density function of $\tilde{X}$ which is a function of the joint probability density distribution of the variables $\tilde{X}_D$, $\tilde{X}_T$, $\tilde{X}_{CL}$. Further, it is equal to the probability density function of FR. $\Phi(\tilde{X})$ is defined as the cumulative probability distribution function of

$\tilde{X}$, which is equal to the cumulative probability distribution function of FR.

From equation (4), given $\overline{D}$, $\overline{T}$, and $\overline{CL}$, the anticipated level of the free reserves $\overline{FR}$ will be controlled by the decision variable I. Any optimum level of I will optimize $\overline{FR}$. For a representative bank J, (J is used as a subscript for variables) it is true that

$$\frac{d\overline{FR}_J}{dI_J} = -1,$$

which is equal to the small change in $\overline{FR}_J$ due to small change of I_J.[1] Bank J tries to maximize its expected return from its portfolio. It is not holding surplus reserves and borrowing reserves at the same time. Therefore, borrowing can be identified by $-FR_J$ and surplus reserves by $+FR_J$.

Bank J's expected return (expected profit) can be written as follows:

$$P_J = K_J + r_{CL}\overline{CL}_J + iI_J + r_s\overline{FR}_J + (r - r_s)\int_{-\infty}^{-\overline{FR}_J} (\overline{FR}_J + \tilde{X}_J)\, d\Phi(\tilde{X}_J) \qquad (6)$$

where r_{CL} = Rate of return on CL

i = Rate of return on I

r_s = Rate of return on holding surplus

r = Rate of cost on borrowed funds

K_J = A component of profit which is independent of portfolio decision

P_J = Expected return or expected profit.

---

[1]This is true under the assumption that bank J is a small component of the banking system. Bank J does not anticipate its sales or acquisitions of assets to affect its deposits significantly.

From FOC, for the profit maximization procedure for bank J, the optimum values of $\overline{FR}_J$ and I_J are derived.

$$\overline{FR}_J^* = -\Phi^{-1}\left(\frac{i-r_s}{r-r_s}\right) \tag{7}$$

$$I_J^* = \overline{D}_J(1-\delta)+\overline{T}_J(1-\tau)-\overline{CL}_J+CA_J+\Phi^{-1}\left(\frac{i-r_s}{r-r_s}\right) \tag{8}$$

where

$\overline{FR}_J$ = The optimum level for the mathematical expectations of FR for bank J;

I_J^* = The optimum level for bank J's investment portfolio.

Assuming all banks face the same interest rates and the same reserves requirement ratios, the optimum level of the aggregate $\overline{FR}$ and the aggregate I for the banking system is the summation of $\overline{FR}_J^*$ and I_J^*, respectively, over all bank J's. Therefore, the aggregate of $\overline{FR}_J^*$ and the aggregate of I_J^* at time t, in the banking system is written as follows:

$$\overline{FR}_t^* = -\psi\left(\frac{i-r_s}{r-r_s}\right) \tag{9}$$

$$I_t^* = \overline{D}_t(1-\delta)+\overline{T}_t(1-\tau)-\overline{CL}_t+CA_t-\overline{FR}_t^* \quad , \tag{10}$$

assuming the functional form ψ exhibits the same properties as Φ_J^{-1} .

Additional assumptions are made for the purpose of generalizing the equations (9) and (10) in order to explain the reality better.

1) The investment decision is responsive to the errors in the anticipation of the levels of the portfolios D, T, and CL, at the beginning of the period, in addition to being responsive to the anticipation of the

mentioned variables. This response may not be perfect. So the term $m_Z(Z-\overline{Z})$ is added as an explanatory variable to the investment decision equations where Z = D, T, CL, and m_Z measures the degree of the investment response to the error in the anticipation of variable Z.

2) FR_{t-1} adjusts to its optimum anticipated level in the next period $\overline{FR}^*_t$, not necessarily instantaneously, so the term $\overline{FR}^*_t$ in equation (10) can be replaced by the term $n_F(\overline{FR}^*_t-FR_{t-1})+FR_{t-1}$. The new term states that at time t, the only n_F portion of the difference $(\overline{FR}^*_t-FR_{t-1})$ is added to FR_{t-1}. The equality of $(n_F=1)$ states that FR_{t-1} fully adjusts to its optimum anticipated level in the next period $\overline{FR}^*_t$, at time t. Therefore, n_F measures the speed of adjustment of FR_{t-1} to its optimum anticipated level in the next period $\overline{FR}^*_t$.

3) Banks anticipate the levels of their portfolios based on the past values of them. They are able to forecast the changes from the past values to the current values with some degree of accuracy. So $\overline{Z}_t$ is estimated by the term $Z_{t-1}+m'_Z(Z_t-Z_{t-1})$ where the term m'_Z measures the degree of accuracy in anticipating the level of $\overline{Z}_t$.

Based on the above assumption, the change in the level of the investment portfolio at time t is estimated.

$$\Delta I_t = n_D(1-\delta)\Delta D_t+n_T(1-\tau)\Delta T_t-n_{CL}\Delta CL_t+\Delta CA_t-n_F(\overline{FR}^*_t-FR_{t-1}) \qquad (11)$$

where

$$n_Z = 1-(1-m_Z)(1-m'_Z),\ Z = D,\ T,\ \text{and CL}.$$

Under assumptions for aggregation, the balance sheet for the representative bank J is used for the banking system. The items in bank J's balance sheet are redefined for their aggregate levels. Therefore, equation (2) is also used for the banking system. Assuming the variables T_t and CL_t as exogenous to the system[2] and variable CA_t constant, using equations (2) and (11), the change in the supply of deposit for the banking system at time t is estimated.

$$\Delta D_t = \frac{1}{(1-n_D)+\delta n_D}\Big[\Delta UR+(1-n_{CL})\Delta CL_t - (1-n_T+\tau n_T)\Delta T_t - n_F(\overline{FR}^*_t - FR_{t-1})\Big] \tag{12}$$

The rate of cost of the borrowed funds r is approximated by FFR and is determined by supply and demand for funds in FFM which is expressed as:

$$r_t = R(d_t, FR_t) \tag{13}$$

where

d = FRB's discount rate.

From equation (9), the optimum anticipated level of FR is shown as

$$\overline{FR}^*_t = -\psi\left(1+\frac{i-r}{r-r_s}\right) .$$

Assuming $(r-r_s)$ is relatively constant, then $\overline{FR}^*$ is rewritten as

$$\overline{FR}^*_t = F(i, r) . \tag{14}$$

From equations (12), (13), and (14), FR_t is derived.

---

[2]At least for the purpose of short-term behavior of the banking system.

$$FR_t = \Gamma F\left[i, R(d, FR_t)\right] + (1-\Gamma)FR_{t-1} + \frac{\Gamma}{n_F}\left[(1-n_D)\right.$$

$$\left.\frac{1-\delta}{\delta}\Delta UR_t - (1-n_{CL})\Delta CL_t + n_T^*\Delta T_t\right] \tag{15}$$

where

$$\Gamma = \frac{n_F}{(1-n_D)+\delta n_D}$$

$$n_T^* = (1-n_T)+\tau(n_T-n_D) - \frac{\tau}{\delta}(1-n_D).$$

The functional form F is linearly approximated.

$$FR_t = a_0 + a_1 i + a_2 d + a_3(1-\Gamma)FR_{t-1} + a_3\frac{\Gamma}{n_F}\left[(1-n_D)\,\frac{1-\delta}{\delta}\,\Delta UR_t\right.$$

$$\left.-(1-n_{CL})\Delta CL_t + n_t^*\Delta T_t\right]. \tag{16}$$

The decision period might be considered as one week, but quarterly data are used so the time dimension must be changed. Assuming m decision periods in each quarter[3] then equation (16) might be rewritten as

$$FR(t) = \sum_{j=1}^{m}\left[a_3(1-\Gamma)\right]^{m-j}\left\{a_0 + a_1 i(t,j) + \right.$$

$$a_2 d(t,j) + \frac{a_3\Gamma}{n_F}\left[(1-n_D)\,\frac{1-\delta}{\delta}\,\Delta UR(t,j)\right.$$

$$\left.\left.-(1-n_{CL})\Delta CL(t,j) + n_T^*\Delta T(t,j)\right]\right\}$$

$$+\left[a_3(1-\Gamma)\right]^{m}\cdot FR(t-1) \tag{17}$$

where FR(t) = FR at the end of the period t

FR(t-1) = FR at the end of the period t-1, or at the beginning of the period t

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[3] m might be equal to thirteen.

$Z(t,j)$ = value of the variable Z at decision period j at quarter t.

t has been redefined in terms of a period of one quarter. Assuming values of i and d are constant over decision period j and $\Delta Z(t,j) = \frac{\Delta Z(t)}{m}$, which means the value of ΔZ at decision period j at quarter t is $\frac{1}{m}$th of the value of ΔZ at quarter t, then equation (17) is rewritten as:

$$\begin{aligned} FR_t = & \sum_{j=1}^{m}\left[a_3(1-\Gamma)\right]^{m-j} a_0 + \sum_{j=1}^{m}\left[a_3(1-\Gamma)\right]^{m-j} a_1 i_t \\ & + \sum_{j=1}^{m}\left[a_3(1-\Gamma)\right]^{m-j} a_2 d_t + \frac{a_3\Gamma}{n_F}(1-n_D)\frac{1-\delta}{\delta}\cdot m \\ & \cdot \sum_{j=1}^{m}\left[a_3(1-\Gamma)\right]^{m-j} \Delta UR_t - \frac{a_3\Gamma}{n_F}(1-n_{CL})\cdot m \\ & \cdot \sum_{j=1}^{m}\left[a_3(1-\Gamma)\right]^{m-j} \Delta CL_t + \frac{a_3\Gamma}{n_F} n_T^{*}\cdot m \cdot \sum_{j=1}^{m}\left[a_3(1-\Gamma)\right]^{m-j} \\ & \cdot \Delta T_t + \left[a_3(1-\Gamma)\right]^{m} FR_{t-1} \quad . \end{aligned} \tag{18}$$

Equation (18) is empirically estimated. For the purpose of money supply determination, the identity of

$$D_t \equiv \frac{UR_t - \tau T_t - FR_t}{\delta} \quad \text{under CRA is utilized.}$$

The inclusion of "lagged reserve accounting" (Model I under lagged reserve accounting)

The results presented by Modigliani, Rasche, and Cooper can be altered and modified by inclusion of LRA in the individual bank's behavioral functions, the equation of the banking system and the process of money supply determination.

Representative Bank's Balance Sheet

Assets			Liabilities	
Reserve total		R_t	Demand Deposits	D_t
Required	RR_t		Federal Government	D_{g_t}
against demand deposits of two periods ago	δD_{t-2}		All others	D_{p_t}
			Time deposits	T_t
against time deposits of two periods ago	τT_{t-2}		Borrowing (from FRB and FFM)	B^*_t
Surplus (excess reserves + loans in FFM)	S_t		Miscellaneous liabilities and Capital	CA_t+MA_t
Commercial loans		CL_t		
Other Investment		I_t		
Miscellaneous Assets		MA_t		

t stands for period of one week.

From the representative bank's balance sheet, the following identities can be derived:

$$UR_t \equiv R_t - B^*_t \tag{19}$$

$$FR_t \equiv S_t - B^*_t = UR_t - RR_t \tag{20}$$

$$FR_t \equiv D_t + T_t - RR_t - CL_t - I_t + CA_t \tag{21}$$

$$FR_t \equiv D_t - \delta D_{t-2} + T_t - \tau T_{t-2} - CL_t - I_t + CA_t \tag{22}$$

where

$$RR_t \equiv \delta D_{t-2} + \tau T_{t-2} \quad . \tag{23}$$

Based on the same assumptions of the portfolios of the representative bank as seen in Model I under CRA, it can be written:

$$D_t = \overline{D}_t + \tilde{X}_{D_t}$$

$$T_t = \overline{T}_t + \tilde{X}_{T_t}$$

$$CL_t = \overline{CL}_t + \tilde{X}_{CL_t}$$

where $\overline{D}_t$, $\overline{T}_t$, and $\overline{CL}_t$ are the expected values of the variables and $\tilde{X}_{D_t}$, $\tilde{X}_{T_t}$, and $\tilde{X}_{CL_t}$ are the pure random parts with expected values of zero.

$$E(\tilde{X}_{D_t}) = E(\tilde{X}_{T_t}) = E(\tilde{X}_{CL_t}) = 0.$$

In contrast to CRA, RR_t under LRA is not a random variable and it is derived from the past observations of D and T. Equation (21) can be rewritten as follows:

$$FR_t = \overline{D}_t + \tilde{X}_{D_t} + \overline{T}_t + \tilde{X}_{T_t} - RR_t - \overline{CL}_t - \tilde{X}_{CL_t} - I_t + CA_t$$

or

$$FR_t = \left[\overline{D}_t + \overline{T}_t + \overline{CL}_t - RR_t - I_t + CA_t\right] + \left[\tilde{X}_{D_t} + \tilde{X}_{T_t} - \tilde{X}_{CL_t}\right] \; .$$

Define:

$$\overline{FR}_t = \overline{D}_t + \overline{T}_t + \overline{CL}_t - RR_t - I_t + CA_t \tag{24}$$

$$\tilde{X}_t = \tilde{X}_{D_t} + \tilde{X}_{T_t} - \tilde{X}_{CL_t} . \tag{25}$$

Then:

$$FR_t = \overline{FR}_t + \tilde{X}_t \tag{26}$$

where $\overline{FR}_t$ = The expected value for FR_t

$\tilde{X}_t$ = Pure random part of FR_t with the expected value of zero $E(\tilde{X}_t) = 0$.

Define:

$g(\tilde{X}_t)$ = The probability density function of $\tilde{X}_t$ (or of FR) for the representative bank.

$G(\tilde{X}_t)$ = The cumulative probability distribution function of $\tilde{X}_t$ (or of FR) for the representative bank.

The representative bank J (J is used as a subscript for the variables) has known anticipations for T_{J_t}, D_{J_t}, and CL_{J_t}. The value of RR_{J_t} is known and CA_{J_t} is assumed to be constant. Under these conditions, from equation (24), the value of $\overline{FR}_{J_t}$ would be controlled by the decision variable I_{J_t}. Any optimized level of I_{J_t} would optimize $\overline{FR}_{J_t}$. Assuming bank J is a small component of the banking system, then from equation (24) it can be written:

$$\frac{d\overline{FR}_{J_t}}{dI_{J_t}} = -1.$$

Assuming that bank J withholds both borrowing and surplus at the same time, that demand by the nonbank-public for commercial loans are exogenous to bank J's decision process. The expected return then, of representative bank J at time t, can be formulated as follows:

$$P_{J_t} = K_{J_t} + r_{CL}\overline{CL}_{J_t} + iI_{J_t} + r\int_{-\infty}^{0} FR_{J_t} g(FR_{J_t}) dFR_{J_t}$$

$$+ r_s \int_{0}^{+\infty} FR_{J_t} g(FR_{J_t}) dFR_{J_t} \qquad (27)$$

where

$r_{CL}\overline{CL}_{J_t}$ = The expected return on CL_{J_t};

iI_{J_t} = The expected return on I_{J_t} .

$r\int_{-\infty}^{0} FR_{J_t} g(FR_{J_t}) dFR_{J_t}$ = The expected cost of borrowing (when bank J is in a situation of borrowing reserves or having a negative FR_{J_t}).

$r_s \int_{0}^{+\infty} FR_{J_t} g(FR_{J_t}) dFR_{J_t}$ = The expected return of having a surplus (when bank J has a surplus of reserves or has a positive FR_{J_t}).

From equation (26) it is shown that:

$$d\tilde{X}_{J_t} = dFR_{J_t},$$

and the range for random variable $\tilde{X}_{J_t}$ is $(-\overline{FR}_{J_t}, -\infty)$.

Therefore, equation(27) can be rewritten as:

$$P_{J_t} = K_{J_t} + r_{CL}\overline{CL}_{J_t} + iI_{J_t} + r\int_{-\infty}^{-\overline{FR}_{J_t}} (\tilde{X}_{J_t} + \overline{FR}_{J_t}) \cdot$$

$$g(\tilde{X}_{J_t})d\tilde{X}_{J_t} + r_s\int_{-\overline{FR}_{J_t}}^{+\infty} (\tilde{X}_{J_t} + \overline{FR}_{J_t}) \cdot$$

$$g(\tilde{X}_{J_t}) \cdot d\tilde{X}_{J_t} \quad .$$

The term

$$r_s\int_{-\infty}^{-\overline{FR}_{J_t}} (\tilde{X}_{J_t} + \overline{FR}_{J_t}) \cdot g(\tilde{X}_{J_t}) \cdot d\tilde{X}_{J_t}$$

will be added and subtracted to the above equation. Thus,

$$P_{J_t} = K_{J_t} + r_{CL}\overline{CL}_{J_t} + iI_{J_t} + (r-r_s)\int_{-\infty}^{-\overline{FR}_{J_t}} (\tilde{X}_{J_t} + \overline{FR}_{J_t}) \cdot$$

$$g(\tilde{X}_{J_t})d\tilde{X}_{J_t} + r_s \left[\int_{-\infty}^{-\overline{FR}_{J_t}} (\tilde{X}_{J_t} + \overline{FR}_{J_t}) g(\tilde{X}_{J_t}) \right. \quad .$$

$$d\tilde{X}_{J_t} + \int_{-\overline{FR}_{J_t}}^{+\infty} (\tilde{X}_{J_t} + \overline{FR}_{J_t}) \cdot g(\tilde{X}_{J_t}) \cdot d\tilde{X}_{J_t} \Bigg] .$$

The bracketed term in the above equation can be rewritten in the following way:

$$\text{Bracketed term} = \int_{-\infty}^{+\infty} (\tilde{X}_{J_t} + \overline{FR}_{J_t}) g(\tilde{X}_{J_t}) d\tilde{X}_{J_t}$$

$$= \left[\int_{-\infty}^{+\infty} \tilde{X}_{J_t} g(\tilde{X}_{J_t}) d\tilde{X}_{J_t} \right] + \left[\overline{FR}_{J_t} \cdot \int_{-\infty}^{+\infty} g(\tilde{X}_{J_t}) \cdot d\tilde{X}_{J_t} \right].$$

It is also known that:

$$\int_{-\infty}^{+\infty} \tilde{X}_{J_t} g(\tilde{X}_{J_t}) d\tilde{X}_{J_t} = E(\tilde{X}_{J_t})$$

$$\int_{-\infty}^{+\infty} g(\tilde{X}_{J_t}) d\tilde{X}_{J_t} = 1.$$

So the bracketed term will be equal to $E(\tilde{X}_{J_t}) + \overline{FR}_{J_t} \cdot 1$.

Since $E(\tilde{X}_{J_t}) = 0$, the bracketed term is equal to $\overline{FR}_{J_t}$. Moreover, it is known that

$$dG(\tilde{X}_{J_t}) = g(\tilde{X}_{J_t}) d\tilde{X}_{J_t} .$$

Finally, equation (27) can be rewritten as:

$$P_{J_t} = K_{J_t} + r_{CL}\overline{CL}_{J_t} + iI_{J_t} + (r-r_s)\int_{-\infty}^{-\overline{FR}_{J_t}} (\tilde{X}_{J_t} + \overline{FR}_{J_t}) \cdot$$

$$dG(\tilde{X}_{J_t}) + r_s\overline{FR}_{J_t}. \tag{28}$$

For the profit maximizing procedure, we will differentiate the profit function with respect to the decision variable I_t:

$$\frac{\partial P_{J_t}}{\partial I_{J_t}} = i_t + (r-r_s) \cdot \frac{\partial(-\overline{FR}_{J_t})}{\partial I_{J_t}} \cdot \lim_{A\to-\infty} \left[\int_A^{-\overline{FR}_{J_t}} \frac{\partial(\overline{FR}_{J_t} + \tilde{X}_{J_t})}{\partial(-\overline{FR}_{J_t})} \cdot \right.$$

$$dG(\tilde{X}_{J_t}) + (\overline{FR}_{J_t} - \overline{FR}_{J_t})dG(-\overline{FR}_{J_t}) \cdot \frac{\partial(-\overline{FR}_{J_t})}{\partial(-\overline{FR}_{J_t})}$$

$$\left. - (\overline{FR}_{J_t} + A)dG(A) \cdot \frac{\partial(A)}{\partial(-\overline{FR}_{J_t})}\right] + r_s \cdot \frac{\partial(-\overline{FR}_{J_t})}{\partial I_{J_t}}$$

where A is an arbitrary variable,

$$\frac{\partial(-\overline{FR}_{J_t})}{\partial I_{J_t}} = 1, \quad \frac{\partial \overline{FR}_{J_t}}{\partial I_{J_t}} = -1, \quad \text{and} \quad \frac{\partial(\overline{FR}_{J_t} + \tilde{X}_{J_t})}{\partial(-\overline{FR}_{J_t})} = -1.$$

From probability distribution for FR_t, it can be written:

$$G(-\infty) = 0 \text{ and } dG(-\infty) = 0, \text{ or } \lim_{A\to-\infty} dG(A) = 0.$$

Therefore,

$$\frac{\partial P_{J_t}}{\partial I_{J_t}} = i-(r-r_s) \cdot \lim_{A\to-\infty} \left[\int_A^{-\overline{FR}_{J_t}} dG(\tilde{X}_{J_t})\right] -r_s$$

or

$$\frac{\partial P_{J_t}}{\partial I_{J_t}} = i - (r-r_s) \cdot \lim_{A \to -\infty} \left[G(-\overline{FR}_{J_t}) - G(A) \right] - r_s \ .$$

Thus,

$$\frac{\partial P_{J_t}}{\partial I_{J_t}} = i - (r-r_s)G(-\overline{FR}_{J_t}) - r_s \ . \tag{29}$$

For FOC:

$$\frac{\partial P_{J_t}}{\partial I_{J_t}} = 0,$$

so

$$i - (r-r_s)G(-\overline{FR}_{J_t}) - r_s = 0$$

or

$$G(-\overline{FR}_{J_t}) = \frac{i-r_s}{r-r_s} \ .$$

Accordingly, the optimum level of $\overline{FR}_{J_t}$ for bank J can be written as

$$\overline{FR}^*_{J_t} = -G^{-1}_{J_t}\left(\frac{i-r_s}{r-r_s}\right) \ . \tag{30}$$

$G(\overline{FR}_{J_t})$ is a monotonically increasing function. Thus, $-G(\overline{FR}_{J_t})$ would be a monotonically decreasing function. As seen above, as i increases, $G(-\overline{FR}_{J_t})$ would increase, so $-G^{-1}_{Jt}\left(\frac{i-r_s}{r-r_s}\right)$ would decrease.

$$\frac{\partial \overline{FR}^*_{J_t}}{\partial i} < 0.$$

Therefore, it follows:

$$\frac{\partial I^*_{J_t}}{\partial i} = -\frac{\partial \overline{FR}^*_{J_t}}{\partial i} > 0.$$

For a similar reason

$$\frac{\partial \overline{FR}^{*}_{J_t}}{\partial r} < 0.$$

Therefore, one can write:

$$\frac{\partial I^{*}_{J_t}}{\partial r} = - \frac{\partial \overline{FR}^{*}_{J_t}}{\partial r} < 0.$$

By substituting the value of $\overline{FR}^{*}_{J_t}$ from equation (30) for $\overline{FR}_{J_t}$ in equation (24), the optimum level of investment decision for bank J under LRA will be derived.

$$I^{*}_{J_t} = \overline{D}_{J_t} + \overline{T}_{J_t} - \overline{CL}_{J_t} - RR_{J_t} + CA_{J_t} + G^{-1}_{J_t}\left(\frac{i-r_s}{r-r_s}\right) \quad . \tag{31}$$

For SOC of profit maximization, one can write:[1]

$$\frac{\partial^2 P_{J_t}}{\partial I^2_{J_t}} = - (r-r_s) \frac{\partial G(-\overline{FR}_{J_t})}{\partial(-\overline{FR}_{J_t})} \cdot \frac{\partial(-\overline{FR}_{J_t})}{\partial I_{J_t}}$$

$$= - (r-r_s) \cdot g(-\overline{FR}_{J_t}).$$

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[1]The comparative static results can also be derived from SOC.

$$\frac{\partial \overline{FR}^{*}_{J_t}}{\partial i} = - \frac{\dfrac{\partial^2 P_{J_t}}{\partial I_{J_t} \partial i}}{\dfrac{\partial^2 P_{J_t}}{\partial I_{J_t} \partial \overline{FR}^{*}_{J_t}}} = - \frac{1}{-(r-r_s) \dfrac{\partial G(-\overline{FR}_{J_t})}{\partial(-\overline{FR}_{J_t})} \cdot \dfrac{\partial(-\overline{FR}_{J_t})}{\partial(\overline{FR}_{J_t})}} \quad .$$

(Footnote continued on following page).

Moreover:

$$g(-\overline{FR}_{J_t}) > 0, \quad r > 0, \quad r_s > 0.$$

So the condition for the profit maximization would be:

$$\frac{\partial^2 P_{J_t}}{\partial I^2_{J_t}} < 0, \quad \text{or } -(r-r_s) < 0, \quad \text{or} \quad r > r_s.$$

(Footnote continued from preceding page).

We have:

$$\frac{\partial G(-\overline{FR}_{J_t})}{\partial(-\overline{FR}_{J_t})} > 0, \quad \frac{\partial(-\overline{FR}_{J_t})}{\partial(\overline{FR}_{J_t})} = -1,$$

and from SOC $-(r-r_s) < 0$.

Thus,

$$\frac{\partial(\overline{FR}^*_{J_t})}{\partial i} < 0.$$

Furthermore,

$$\frac{\partial \overline{FR}^*_{J_t}}{\partial r} = -\frac{\dfrac{\partial^2 P_{J_t}}{\partial I_{J_t} \cdot \partial r}}{\dfrac{\partial^2 P_{J_t}}{\partial I_{J_t} \cdot \partial \overline{FR}^*_{J_t}}} = -\frac{-G(-\overline{FR}_{J_t})}{\dfrac{\partial^2 P_{J_t}}{\partial I_{J_t} \cdot \partial \overline{FR}^*_{J_t}}}.$$

It is known that

$$-G(-\overline{FR}_{J_t}) < 0, \quad \frac{\partial^2 P_{J_t}}{\partial I_{J_t} \cdot \partial \overline{FR}^*_{J_t}} > 0.$$

So

$$\frac{\partial(\overline{FR}^*_{J_t})}{\partial r} > 0.$$

On the other hand, $G(-\overline{FR}_{J_t})$ is a cumulative probability distribution so it must be positive. Therefore, from FOC, it can be stated that

$$\frac{i-r_s}{r-r_s} > 0, \quad \text{or} \quad i > r_s.$$

In order for the optimum levels of $\overline{FR}_{J_t}$ and I_{J_t} to exist and result in the maximum level of profit, the following condition must be satisfied:

$$i > r > r_s.$$

Assuming that all banks face the same interest rates in all markets and the same required reserve ratios, we can aggregate $\overline{FR}^*_{J_t}$ and $I^*_{J_t}$ over all banks. The optimum level of the expected free reserves and the optimum level of investment in the system as a whole can be derived as follows:

$$\overline{FR}^*_t = \sum_{J=1}^{n_0} \overline{FR}^*_{J_t} = \sum_{J=1}^{n_0} - G^{-1}_{J_t} \left(\frac{i-r_s}{r-r_s}\right) .$$

Therefore,

$$\overline{FR}^*_t = -H_t\left(\frac{i-r_s}{r-r_s}\right) \tag{32}$$

and

$$I^*_t = \sum_{J=1}^{n_0} I^*_{J_t} = \sum_{J=1}^{n_0} \overline{D}_{J_t} + \sum_{J=1}^{n_0} \overline{T}_{J_t} - \sum_{J=1}^{n_0} \overline{CL}_{J_t}$$

$$- \sum_{J=1}^{n_0} RR_{J_t} + \sum_{J=1}^{n_0} CA_{J_t} + \sum_{J=1}^{n_0} G^{-1}_{J_t} \left(\frac{i-r_s}{r-r_s}\right) .$$

It follows

$$I^*_t = \overline{D}_t + \overline{T}_t - \overline{CL}_t - RR_t + CA_t + H_t \left(\frac{i-r_s}{r-r_s}\right) \tag{33}$$

where the H function exhibits the same properties as G^{-1}_J .

n_0 = the number of the banks in the banking system.

In a more realistic manner, one can assume the following:

1) Investment decision responds to the errors in anticipations besides being responsive to the levels of the anticipations of the variables;

2) Adjustment of the FR to its optimum is slow. Then, under LRA, the optimum level of investment decision equation can be rewritten as:

$$I_t^* = \overline{D}_t + m_D(D_t - \overline{D}_t) + \overline{T}_t + m_T(T_t - \overline{T}_t) - \overline{CL}_t - m_{CL}(CL_t - \overline{CL}_t)$$

$$+ CA_t - RR_t - \left[n_F(\overline{FR}_t^* - FR_{t-1}) + FR_{t-1} \right]$$

where

m_Z (= m_D, m_T, and m_{CL}) measures the degree of investment responsiveness to the errors in anticipation of the variable Z (= D, T, and CL).

Assuming that the anticipations of the variable Z can be based on its past values, it can then be stated that

$$\overline{Z}_t = Z_{t-1} + m'_Z(Z_t - Z_{t-1}) \quad \text{or} \quad (Z_t - \overline{Z}_t) = (1-m'_Z)(Z_t - Z_{t-1})$$

where m'_Z measures the degree of accuracy in anticipating the level of $\overline{Z}_t$.

So:

$$\overline{Z}_t + m_Z(Z_t - \overline{Z}_t) = - \left[(Z_t - \overline{Z}_t) - m_Z(Z_t - \overline{Z}_t) - Z_t \right]$$

$$= - \left[(1-m_Z)(Z_t - \overline{Z}_t) - Z_t \right] = + \left[(Z_t - Z_{t-1}) - (1-m_Z)(1-m'_Z)(Z_t - Z_{t-1}) + Z_{t-1} \right] = \left[1-(1-m_Z)(1-m'_Z) \right] \cdot (Z_t - Z_{t-1}) + Z_{t-1} \ .$$

Define

$$n_Z = 1-(1-m_Z)(1-m_Z'),$$

so

$$\overline{Z}_t + m_Z(Z_t-\overline{Z}_t) = n_Z(Z_t-Z_{t-1}) + Z_{t-1},$$

so the equation for the optimum investment decision can be rewritten as

$$I_t^* = n_D(D_t-D_{t-1}) + D_{t-1}+ n_T(T_t-T_{t-1}) + T_{t-1} - n_{CL}(CL_t-CL_{t-1})$$

$$- CL_{t-1} + (CA_t-CA_{t-1}) + CA_{t-1} - (RR_t-RR_{t-1}) - RR_{t-1}$$

$$-n_F(\overline{FR}_t^* - FR_{t-1}) - FR_{t-1},$$

or

$$I_t^* - \left[D_{t-1}+T_{t-1}-CL_{t-1}+CA_{t-1}-RR_{t-1}-FR_{t-1}\right] =$$

$$n_D\Delta D_t+n_T\Delta T_t-n_{CL}\Delta CL_t+\Delta CA_t-\Delta RR_t-n_F(\overline{FR}_t^*-FR_{t-1}).$$

The balance sheet for the representative bank J and identities (21) and (23) can be used for the banking system. Therefore, from equation (21) it can be stated:

$$I_{t-1} = D_{t-1}+T_{t-1}-CL_{t-1}+CA_{t-1}-RR_{t-1}-FR_{t-1}$$

and define

$$\Delta I_t^* = I_t^*-I_{t-1}, \text{ and assume } \Delta I_t = \Delta I_t^*.$$

Therefore,

$$\Delta I_t = n_D\Delta D_t+n_T\Delta T_t-n_{CL}\Delta CL_t+\Delta CA_t-\Delta RR_t-n_F(\overline{FR}_t^*-FR_{t-1}). \qquad (34)$$

Given ΔCA_t, ΔRR_t, FR_{t-1}, and assuming that at least in the short-run, ΔT_t and ΔCL_t are considered to be exogenous variables, then the change in the supply of deposit for the banking system at time t, under LRA, can be estimated by equations (21) and (34). From equation (21), it is known

$$\Delta I_t = \Delta D_t + \Delta T_t - \Delta CL_t + \Delta CA_t - \Delta RR_t - \Delta FR_t.$$

By substituting in equation (34) for ΔI_t, from the above equations, the results are

$$\Delta D_t = -\Delta T_t + \Delta CL_t - \Delta CA_t + \Delta RR_t + \Delta FR_t + n_D \Delta D_t + n_T \Delta T_t - n_{CL} \Delta CL_t + \Delta CA_t -$$
$$\Delta RR_t - n_F(\overline{FR}^*_t - FR_{t-1}).$$

Thus

$$\Delta D_t = \frac{1}{1-n_D}\left[\Delta FR_t - (1-n_T)\Delta T_t + (1-n_{CL})\Delta CL_t - n_F(\overline{FR}^*_t - FR_{t-1})\right] \qquad (35)$$

or

$$\Delta D_t = \frac{1}{1-n_D}\left[\Delta UR_t - (1-n_T)\Delta T_t + (1-n_{CL})\Delta CL_t - n_F(\overline{FR}^*_t - FR_{t-1}) - \Delta RR_t\right] . \qquad (36)$$

Equation (36) relates the variable ΔD_t to the policy variable UR_t, given constant RR_t and FR_{t-1}, exogenous T_t and CL_t, and the optimized level of $\overline{FR}_t(\overline{FR}^*_t)$.

In reality, banks can neither predict the values of the variables and adjust for their errors perfectly, nor have they complete inability of anticipating and adjusting for the variables. Banks can partially anticipate the values of the variables and adjust for the errors in the variables; in other words: $0 < n_D, n_T, n_{CL}, n_F < 1$.

Let's consider the cases in which n_D, n_T, n_{CL}, and n_F take their extreme values.

Assume $n_T = n_{CL} = 1$, which means that banks can perfectly forecast the values of T_t and CL_t and fully adjust for their forecast errors. Moreover, since T_t and CL_t are exogenous, we should see no effect from ΔT_t and ΔCL_t on the change in D_t. As $n_T = n_{CL} = 1$, then $1-n_T = 1-n_{CL} = 0$. Therefore, the terms ΔT_t and ΔCL_t would not appear in the equation for the change in D_t. Equation (36) can be rewritten as

$$\Delta D_t = \frac{1}{1-n_D}\left[\Delta UR_t - n_F(\overline{FR}_t^* - FR_{t-1}) - \Delta RR_t\right].$$

Moreover, assume $n_F = 1$, or FR adjusts to its desired level instantaneously; the results are then

$$\Delta D_t = \frac{1}{1-n_D}\left[\Delta UR_t - (\overline{FR}_t^* - FR_{t-1}) - \Delta RR_t\right].$$

The above equation states that: given ΔRR_t based on the deposits of previous weeks, any change in the policy variable UR_t by the Fed will affect ΔD_t as long as $\overline{FR}_t^*$ is different from FR_t.

If $FR_t > \overline{FR}_t^*$, then the surplus of the free reserves will determine the increase in D_t, and if $FR_t < \overline{FR}_t^*$, the shortage of the free reserves will determine the decrease in D_t.[2]

Moreover, consider the case that only $n_D = 1$, which means the banks can perfectly anticipate the level of the demand deposit; they can

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[2]Note that in the long-run, if we continue to have the surplus of the free reserves, D_t would continuously increase, and vice versa. Consequently, under LRA, the fact that n_T, n_{CL}, or n_F in reality are not equal to one, is very crucial.

accurately forecast any errors of the anticipation, and completely remedy errors. Thus

$$\Delta D_t = \frac{1}{0}\left[\Delta UR_t - (1-n_T)\Delta T_t + (1-n_{CL})\Delta CL_t - n_F(\overline{FR}^*_t - FR_{t-1}) - \Delta RR_t\right] .$$

The change in the deposit is not defined by the changes of UR_t, RR_t, FR_t, or the exogenous variables. The result is not surprising, because D_t is perfectly and accurately forecasted. The optimization of any variables such as FR_t or the changes of any exogenous variables would not affect the full predicted level of D_t. In this case, any change in the policy variable has neither direct nor indirect effect on the change in the level of the deposit.[3]

Then, for the other extreme case, assume $n_D = n_T = n_{CL} = 0$, which means banks are completely unable to forecast and remedy errors in their forecasts of the variables D_t, T_t, CL_t. In this case, the model will be fully affected by the variations in the exogenous variables T_t and CL_t. That is shown by equation (36) when $n_D = n_T = n_{CL} = 0$.

$$\Delta D_t = \Delta UR_t - n_F(\overline{FR}^*_t - FR_{t-1}) - \Delta RR_t - \Delta T_t + \Delta CL_t \quad .$$

The above equation formulates the step-by-step adjustment process which is taken by the banks in response to any change in the policy variables. Assume:

$$n_F = 1, \quad \Delta T = \Delta CL = \overline{FR}^* = 0, \text{ then:}$$

$$\Delta D_t = \Delta UR_t + FR_{t-1} - \Delta RR_t,$$

---

[3]In general, the change in the policy variable due to making a discrepancy between actual and desired level of the free reserves affects indirectly the change in the level of the deposit.

or

$$\Delta D_t = FR_t.$$

These equations state that as the Fed increases its policy variable namely UR, by the amount equal to "A" dollars, then as a first step, the deposit level would increase by the same amount. Then, banks would hold a surplus of reserves or positive free reserves by the amount of "A" dollars.

By the above equations

$$\Delta D_t = A+0-0$$

$$\Delta D_t = A.$$

Assume initially $FR_{t-1}=0$ and $\Delta RR_t=0$.

In the second step, we have $\Delta UR=0$. Banks still hold their surplus equal to the amount of "A" dollars. Because under the LRA system, no required reserves have been applied to the banks' extra deposits. Thus, in the second step by the above equations, it is written:

$$\Delta D_t = 0+A-0$$

$$\Delta D_t = A.$$

By the same reasoning, the third, and subsequent steps would give the same results and ΔD would be equal to the amount of the original change in the policy variable (= "A" dollars).[4]

---

[4]In contrast to LRA under CRA, when we assume $n_D = n_T = n_{CL} = 0$, $n_F = 1$, and $\Delta T = \Delta CL = \overline{FR}^* = 0$, then

$$\Delta D_t = \Delta UR_t + FR_{t-1} \quad \text{or} \quad \Delta D_t = \Delta RR_t + FR_t \quad .$$

In the first step, it is originally known: ΔUR = "A" dollars, then by the above equations, it is shown:

(Footnote continued on following page).

As we have seen in equation (32) under LRA variable $\overline{FR}_t^*$ can be expressed as:

$$\overline{FR}_t^* = -H\left(\frac{i-r_s}{r-r_s}\right) = -H\left(1-\frac{i-r}{r-r_s}\right)$$

where i = The short-term Treasury bill rate;

r = The rate for the cost of the borrowings;

r_s = The rate for the return on holding surplus.

i is considered to be exogenous to the system. r depends upon the supply of and demand for funds in the FFM. It also depends upon the amount of the surplus and borrowings in the banking system. r_s is also a function of the transactions in the FFM and in the banking system. Taking the FFM transactions into account, the r and r_s can be expressed as the functions of i, d, and FR (or UR);

$$\overline{FR}_t^* = \Omega(i_t, r_t) \quad \Omega_i<0,\ \Omega_r>0 \tag{37}$$

$$r_t = R(UR_t, d_t) \quad R_{UR}<0,\ R_d>0 \tag{38}$$

(Footnote continued from preceding page).

$$\Delta D_t = A+0$$

$$\Delta D_t = \delta A+(1-\delta)A.$$

In the second step, we have:

$$\Delta D_t = 0+(1-\delta)A$$

$$\Delta D_t = \delta(1-\delta)A+(1-\delta)^2A$$

and so on.... As it is seen under CRA, as a result of the change in the policy variable UR_t, the change in the deposit level would increase by the same amount as in the first step. Due to the increased required reserves, ΔD_t would still increase in the latter steps, but continuously by a smaller amount than the original increase in the policy variable.

where d = The FRB's discount rate which is considered to be exogenous to the system (See Appendix I, FFM under aggregate reserves instrument).

From equations (36), (37), and (38) we can write:

$$\overline{FR}^{*}_{t} = \Omega\left[i_t, R(UR_t, d_t)\right] = h(i_t, d_t, UR_t) \tag{39}$$

where

$$\frac{\partial \overline{FR}^{*}_{t}}{\partial i_t} = h_i = \Omega_i < 0$$

$$\frac{\partial \overline{FR}^{*}_{t}}{\partial d_t} = h_d = \Omega_r \cdot R_d > 0$$

$$\frac{\partial \overline{FR}^{*}_{t}}{\partial UR_t} = h_{UR} = \Omega_r \cdot R_{UR} < 0.$$

Therefore, equation (36) can be rewritten as:

$$\Delta D_t = \frac{-n_F}{1-n_D} \cdot h(i_t, d_t, UR_t) + \frac{1}{1-n_D} \Delta UR_t - \frac{1-n_T}{1-n_D} \Delta T_t$$

$$+ \frac{1-n_{CL}}{1-n_D} \Delta CL_t + \frac{n_F}{1-n_D} FR_{t-1} - \frac{1}{1-n_D} \Delta RR_t. \tag{40}$$

It can be written:

$$FR_{t-1} \equiv UR_{t-1} - RR_{t-1}$$

$$RR_{t-1} \equiv \delta D_{t-3} + \tau T_{t-3}.$$

Then, equation (40) is rewritten as:

$$\Delta D_t = \frac{-n_F}{1-n_D} h(i_t, d_t, UR_t) + \frac{1}{1-n_D} UR_t - \frac{1-n_F}{1-n_D} UR_{t-1}$$

$$- \frac{1-n_T}{1-n_D} \Delta T_t + \frac{1-n_{CL}}{1-n_D} \Delta CL_t - \frac{1}{1-n_D} RR_t + \frac{1-n_F}{1-n_D} RR_{t-1}, \qquad (41)$$

or

$$\Delta D_t = \frac{-n_F}{1-n_D} h(i_t, d_t, UR_t) + \frac{1}{1-n_D} UR_t - \frac{1-n_F}{1-n_D} UR_{t-1} - \frac{1-n_T}{1-n_D} \Delta T_t$$

$$- \frac{\tau}{1-n_D} T_{t-2} + \frac{\tau(1-n_F)}{1-n_D} T_{t-3} + \frac{1-n_{CL}}{1-n_D} \Delta CL_t$$

$$- \frac{\delta}{1-n_D} D_{t-2} + \frac{\delta(1-n_F)}{1-n_D} D_{t-3}.[5] \qquad (42)$$

---

[5]Under the system of LRA, one can write the following identity:

$$D_{t-2} \equiv \frac{UR_t - FR_t - \tau T_{t-2}}{\delta}.$$

It is not feasible to estimate the level of FR_t for the purpose of estimating the level of ΔD_t using the above identity. Under the LRA system, there is a direct and one-to-one relationship between UR_t and FR_t at time t. As the Fed sets the level of UR_t, then the level of FR_t is also determined given RR_t;

$$FR_t \equiv UR_t - (\delta D_{t-2} + \tau T_{t-2}).$$

Therefore, we are able to estimate ΔD_t directly based on the policy variable given the exogenous variables. It is possible however, to estimate the level of FR_t based on the policy variable, the lagged exogenous variables, and the lagged values of deposits, interest rates, and FR. The estimated FR cannot be used for the purpose of estimating the level or change of deposit. Thus, it can be written:

$$\Delta FR_t = \frac{\delta n_F}{1-n_D} h(i_{t-2}, d_{t-2}, UR_{t-2}) - \frac{\delta}{1-n_D} FR_{t-2} + \frac{\delta(1-n_F)}{1-n_D} FR_{t-3}$$

(Footnote continued on following page).

Define

$$\Gamma = \frac{n_F}{1-n_D}\ .$$

Then, equation (42) can be rewritten as:

$$\Delta D_t = -\Gamma h(i_t, d_t, UR_t) + \Gamma/n_F UR_t - \Gamma/n_F(1-n_F)UR_{t-1}$$

$$-\Gamma/n_F(1-n_T)\Delta T_t - \Gamma/n_F\tau\cdot T_{t-2} + \Gamma/n_F\tau\cdot(1-n_F)T_{t-3}$$

$$+\Gamma/n_F(1-n_{CL})\Delta CL_t - \Gamma/n_F\delta D_{t-2} + \Gamma/n_F\cdot\delta\cdot(1-n_F)D_{t-3}. \qquad (43)$$

The following specifications about the signs and magnitudes of the coefficients of equations (40), (41), (42) can be made. In reality, the ability of the banks to accurately forecast and adjust the errors in the anticipation for D_t is very restricted. So, n_D is small, and close to zero, and as we have seen:

$0<n_F,\ n_T,\ n_{CL}<1, \quad 0<\delta,\tau<1.$ It can therefore be written:

$$\frac{1}{1-n_D} \geq 1, \qquad 0 < \Gamma = \frac{n_F}{1-n_D} < 1,\ 0 < 1-n_F < 1, \qquad 0 < 1-n_T < 1,$$

$$0 < 1-n_{CL} < 1.$$

(Footnote continued from preceding page).

$$+\ \Delta UR_t + \left[\frac{\delta(1-n_T)-\tau(1-n_D)}{1-n_D}\right]\Delta T_{t-2} - \left[\frac{\delta(1-n_{CL})}{1-n_D}\right]\Delta CL_{t-2}\ .$$

In equation (43) we will have:

$$-1 < \text{Coeff. of } UR_{t-1} = -\Gamma/n_F\,(1-n_F) < 0,$$

$$-1 < \text{Coeff. of } \Delta T_t = -\Gamma/n_F(1-n_T) < 0,$$

$$-1 < \text{Coeff. of } T_{t-2} = -\Gamma/n_F \cdot \tau < 0,$$

$$0 < \text{Coeff. of } T_{t-3} = +\frac{\Gamma}{n_F}\,\tau(1-n_F) < 1,$$

$$0 < \text{Coeff. of } \Delta CL_t = \frac{\Gamma}{n_F}\,(1-n_{CL}) < 1,$$

$$-1 < \text{Coeff. of } D_{t-2} = -\frac{\Gamma}{n_F}\,\delta < 0,$$

$$0 < \text{Coeff. of } D_{t-3} = \frac{\Gamma}{n_F}\,(1-n_F) < 1.$$

In equation (41) we can specify the coefficients of other variables as follows:

$$\text{Coeff. of } RR_t = \frac{-\Gamma}{n_F} \leq -1,$$

$$0 < \text{Coeff. of } RR_{t-1} = \frac{\Gamma}{n_F}\,(1-n_F) < 1.$$

Similarly in equation (40) we have:

$$0 < \text{Coeff. of } FR_{t-1} = \Gamma < 1,$$

$$\text{Coeff. of } \Delta RR_t = -\frac{\Gamma}{n_F} \leq -1.$$

From equation (39) we have: $h_i = \Omega_i < 0$, $h_d = \Omega_r R_d > 0$, and $h_{UR} < 0$.

So from equation (41) or (43) results:

$$\frac{\partial(\Delta D_t)}{\partial d_t} = -\Gamma h_d < 0$$

$$\frac{\partial(\Delta D_t)}{\partial i_t} = -\Gamma h_i > 0$$

$$\frac{\partial(\Delta D_t)}{\partial UR_t} = \frac{\Gamma}{n_F} - \Gamma h_{UR} > 0.$$

From equation (37) we have: $|\Omega_r| \simeq |\Omega_i|$ and from equation (38), in reality, we expect: $R_d < 1$.[6] So: $|h_d| < |h_i|$ or

$$\left|\frac{\partial(\Delta D_t)}{\partial d_t}\right| < \left|\frac{\partial(\Delta D_t)}{\partial i_t}\right|$$ or the change in deposit supply is more responsive to the change in i_t than to d_t.

For the purpose of fitting the deposit supply equations and estimating the coefficients, we will use a linear approximation of equation (30) within a reasonable range of (i_t-d_t). So equation (39) can be rewritten as:

$$\overline{FR}^*_t = a_0+a_1 i_t+a_2 d_t+a_3 UR_t.$$

So equation (43) can be written as:

$$\Delta D_t = -\Gamma a_0-\Gamma a_1 i_t-\Gamma a_2 d_t+(\frac{\Gamma}{n_F} - \Gamma a_3)UR_t - \frac{\Gamma}{n_F}(1-n_F)UR_{t-1}$$

$$- \frac{\Gamma}{n_F}(1-n_T)\Delta T_t - \frac{\Gamma}{n_F}\tau T_{t-2} + \frac{\Gamma}{n_F}\tau(1-n_F)T_{t-3}$$

---

[6]We anticipate b_d to be high due to reluctance of banks for borrowing from FRB's discount window and L_r to be high due to efficient transformation of funds to and from the FFM. See Appendix I, FFM under aggregate reserves instrument.

$$+ \frac{\Gamma}{n_F} (1-n_{CL}) \Delta CL_t - \frac{\Gamma}{n_F} \delta D_{t-2} + \frac{\Gamma}{n_F} \cdot \delta \cdot (1-n_F) \cdot D_{t-3}. \tag{44}$$

Equation (44) is the reduced form for the change in deposit supply which is empirically estimated in Part II of Chapter IV. In specifying the signs of the coefficients for the interest rates and the policy variable UR_t, we can write:

$$a_1 \approx h_i < 0, \quad a_2 \approx h_d > 0, \quad a_3 \approx h_{UR} < 0.$$

Thus:

$$\text{Coeff. of } i_t = - \Gamma a_1 > 0$$

$$\text{Coeff. of } d_t = - \Gamma a_2 < 0$$

$$\text{Coeff. of } UR_t = \frac{\Gamma}{n_F} - \Gamma a_3 > 0.$$

It is also known:

$$|a_1| > |a_2|$$

so

$$|\text{Coeff. of } i_t| > |\text{Coeff. of } d_t|.$$ [7]

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[7] Using an alternative form for finding the FFR from the FFM (see Appendix I, FFM under aggregate reserves instrument) we can get an alternative equation for equation (44). We had:

$$r_t = R^a(FR_t, d_t),$$

$$\overline{FR}^*_t = \Omega(i_t, r_t).$$

So: $$\overline{FR}^*_t = \Omega\left[i_t, R_a(FR_t, d_t)\right] = h^a(i_t, d_t, FR_t).$$

With linear approximation, one can write:

(Footnote continued on following page).

(Footnote continued from preceding page).

$$\overline{FR}_t^* = \alpha_0+\alpha_1 i_t+\alpha_2 d_t+\alpha_3 FR_t,$$

where

$$\alpha_3 \simeq \frac{\partial \overline{FR}_t^*}{\partial FR_t} = \Omega_r R^a{}_{FR} < 0,$$

$$\alpha_2 \simeq \frac{\partial \overline{FR}_t^*}{\partial d_t} = \Omega_r R^a{}_d > 0,$$

$$\alpha_1 \simeq \frac{\partial \overline{FR}_t^*}{\partial i_t} = \Omega_i < 0.$$

Under the system of LRA, we have:

$$FR_t = UR_t - \delta D_{t-2} - \tau T_{t-2},$$

thus:

$$\overline{FR}_t^* = \alpha_0+\alpha_1 i_t+\alpha_2 d_t+\alpha_3 UR_t-\alpha_3\delta D_{t-2}-\alpha_3\tau T_{t-2}.$$

Therefore, equation (44) can be rewritten in the following manner:

$$\Delta D_t = -\Gamma\alpha_0-\Gamma\alpha_1 i_t-\Gamma\alpha_2 d_t+(\frac{\Gamma}{n_F} - \Gamma\alpha_3)UR_t - \frac{\Gamma}{n_R}(1-n_F)UR_{t-1}$$

$$- \frac{\Gamma}{n_F}(1-n_T)\Delta T_t + (\Gamma\alpha_3\tau- \frac{\Gamma}{n_F}\tau)T_{t-2}+ \frac{\Gamma}{n_F}\tau(1-n_F)T_{t-3}$$

$$+ \frac{\Gamma}{n_F}(1-n_{CL})\Delta CL_t+(\Gamma\alpha_3\delta- \frac{\Gamma}{n_F}\delta)D_{t-2}+ \frac{\Gamma}{n_F}\delta(1-n_F)D_{t-3}.$$

As we notice, all signs of the coefficients are the same in the above equation as equation (44). The magnitude of the coefficients for T_{t-2} and D_{t-2} in absolute value are not necessarily less than one.

Part III. Model II

<u>Inclusion of the certificates of deposit transactions in Model I (Model II under contemporaneous reserve accounting)</u>

Large certificates of deposit are commercial bank time deposit liabilities which are issued in denominations of $100,000 or more. Certificates of deposit are either negotiable or nonnegotiable. Negotiable certificates of deposit are those which can be sold in the secondary market. They can be considered as money market instruments. The maturity dates on CD's usually are thirty to ninety days. Like all time deposits, CD's are subject to reserves requirements and FDIC insurance payments. Most CD's are directly sold by banks to investors in the primary market. A small portion of CD's are issued through dealers in the secondary market.

Part II concentrated on commercial-bank investments or earning assets as an instrumental variable for the banks' portfolio management. In reality, and especially within the last twenty years, liability management involving the issuance of CD's has come to be an important instrument for the bank's portfolio adjustment. Studies on the banks' behavior indicate that the banks try to optimize their investments portfolios and, thus, CD liabilities. Then, based on these optimum values, the banks adjust their short-term (one to two weeks) instrumental variables, namely the FFM transactions and/or borrowings from FRB's. In this part it is assumed that the banks choose the decision variables I and CD to maximize the expected level of their profit. The variables $\overline{D}$, $\overline{T}$, and $\overline{CL}$ are known and the variables CA and the reserve ratios are constant with respect to banking behavioral relationships. As a result of the optimization, the desired values for FR and/or supply of deposit are derived.

In Part III, CD transaction is included in Model I from Part II using the CRA system. Individual bank J's balance sheet can be written as follows:

Balance Sheet of Bank J

Assets			Liabilities		
Reserves total		R_J	Demand Deposits		D_J
Required	RR_J		Federal Government	D_{g_J}	
against demand deposit	δD_J		All Others	D_{p_J}	
against time deposit	τT_J		Time Deposits		T_J
against large negotiable certificates of deposit	γCD_J		Large negotiable certificates of deposit		CD_J[1]
			Borrowing		B_J^*
Surplus	S_J		From FRB's	B_J	
excess reserves	ER_J		From FFM	FFB_J	
Loans in FFM	FFL_J		Miscellaneous liabilities and		MA_J+
Commercial loans		CL_J	Capital		CA_J
Other investments		I_J			
Miscellaneous assets		MA_J			

where: γ = required reserve ratio against CD.

The definition of the other variables in the balance sheet of bank J are the same as those seen in Part II.

[1]Bank J is assumed to be a large bank which issues CD's, does not buy the CD's issued by other banks, and does not repurchase its own CD's.

From bank J's balance sheet, it can be written:

$$RR_J \equiv \delta D_J + \tau T_J + \gamma CD_J \tag{45}$$

$$FR_J \equiv D_J + T_J - RR_J - CL_J - I_J + CD_J + CA_J$$

or

$$FR_J \equiv D_J(1-\delta) + T_J(1-\tau) + CD_J(1-\gamma) - CL_J - I_J + CA_J. \tag{46}$$

Taking into account the same set of assumptions in Part II, the outcome of FR_J depends upon the realization of the variables D_J, T_J, and CL_J for any chosen value for I_J and CD_J by profit maximization procedure. Similar to Part II, it can be written that:

$$\begin{aligned} FR_J &= \overline{FR}_J + \tilde{X}_J, \\ \overline{FR}_J &= \overline{D}_J(1-\delta) + \overline{T}_J(1-\tau)-\overline{CL}_J-I_J+CD_J(1-\gamma)+CA_J \\ \tilde{X} &= \tilde{X}_D(1-\delta) + \tilde{X}_T(1-\tau)-\tilde{X}_{CL} \ . \end{aligned} \tag{47}$$

Using equation (47), $\overline{FR}_J$ is controlled by the decision variables I_J and CD_J. The optimization of I_J and CD_J would optimize $\overline{FR}_J$.

From equation (47) it can be written:

$$\frac{\partial \overline{FR}_J}{\partial I_J} = -1, \quad \frac{\partial \overline{FR}_J}{\partial CD_J} = 1 - \gamma.^2$$

Based on the same behavioral assumption as seen in Part II, the bank J's expected profit can be stated.

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[2]Bank J is small component of the banking system.

$$p_J = K_J + r_{CL}\overline{CL}_J + iI_J - \left[r_{CD} + e_J\right] \cdot \; CD_J + r_S\overline{FR}_J$$

$$+ (r - r_s) \int_{-\infty}^{-\overline{FR}_J} (\overline{FR}_J + \tilde{X}_J) d\Phi_J(\tilde{X}_J) \tag{48}$$

where

r_{CD} = the offering rate paid on CD,

e_J = the average subjective cost of issuing CD.

The subjective cost of issuing CD for the representative individual bank J can be explained by the following variables:

1) Outstanding level of CD -

The higher the outstanding level of CD issued by bank J, the higher the subjective cost is for issuing additional CD. As the amount of CD issued becomes larger, the soundness of bank J comes increasingly into question.

2) Amount of commercial loans -

Issuing CD is associated with a high fixed cost (i.e., insurance premiums, required reserves) and no other variable cost besides interest payment on CD. Therefore, bank J uses the funds of issuing CD to finance its commercial loans with a duration of several weeks. This hypothesis has been supported in several studies (see Luckett (29), Depamphilis (13), and Stigum (37)). As the demand for commercial loans increases, banks bear less subjective discomfort or cost for issuing additional CD. CL is also included as an explanatory variable for e.

As a result, it is stated that:

$$e_J = e_J(CD_J, CL_J)$$

where

$$\frac{\partial e_J}{\partial CD_J} > 0, \quad \frac{\partial e_J}{\partial CL_J} < 0.$$

To maximize the expected profit, equation (48) is differentiated with respect to decision variables CD_J and I_J.

$$\begin{cases} \dfrac{\partial P_J}{\partial I_J} = i + (r-r_s)\,\dfrac{\partial(-\overline{FR}_J)}{\partial I_J}\cdot\left[\dfrac{\partial\displaystyle\int_{-\infty}^{-\overline{FR}_J}(\tilde{X}_J+\overline{FR}_J)\,d\Phi_J(\tilde{X}_J)}{\partial(-\overline{FR}_J)}\right] + r_s\cdot\dfrac{\partial\overline{FR}_J}{\partial I_J} \\[2ex] \dfrac{\partial P_J}{\partial CD_J} = -\left[r_{CD}+e_J\right] - \left[\dfrac{\partial e_J}{\partial CD_J}\right]CD_J + (r-r_s)\cdot\dfrac{\partial(-\overline{FR}_J)}{\partial CD_J}\cdot \\[2ex] \qquad\left[\dfrac{\partial\displaystyle\int_{-\infty}^{-\overline{FR}_J}(\tilde{X}_J+\overline{FR}_J)\,d\Phi_J(\tilde{X}_J)}{\partial(-\overline{FR}_J)}\right] + r_s\cdot\dfrac{\partial\overline{FR}_J}{\partial CD_J}\ . \end{cases}$$

Therefore, FOC can be stated as:

$$\begin{cases} i+(r-r_s)\cdot\left[-\Phi(-\overline{FR}_J)\right] - r_s = 0 \\[2ex] -\,(r_{CD}+e_J) - \left(\dfrac{\partial e_J}{\partial CD_J}\right)CD_J - (r-r_s)(1-\gamma)\left[-\Phi_J(-\overline{FR}_J)\right] + r_s(1-\gamma) = 0. \end{cases}$$

Thus

$$\Phi(-\overline{FR}_J) = \frac{i-r_s}{r-r_s}$$

or

$$\overline{FR}_J^{*} = -\Phi_J^{-1}\left(\frac{i-r_s}{r-r_s}\right)\ . \qquad (49)$$

Then it can be written:

$$-(r_{CD}+e_J)-\frac{\partial e_J}{\partial CD_J}\cdot CD_J+(r-r_s)(1-\gamma)\cdot\frac{i-r_s}{r-r_s}+r_s(1-\gamma)=0$$

or

$$-(r_{CD}+e_J)-\frac{\partial e_J}{\partial CD_J}\cdot CD_J+(1-\gamma)i=0$$

or

$$-e_J(CD_J,CL_J)-\frac{\partial e_J}{\partial CD_J}(CD_J,CL_J)\cdot CD_J=r_{CD}-(1-\gamma)i. \qquad (50)$$

Then, the optimum value of CD_J can be stated as

$$CD_J^*=q_J(i,r_{CD},CL_J)\ ^{3} \qquad (51)$$

The SOC for the profit maximization requires that the principal minors of the relevant Hessian determinant alternate in sign, and starting with the first one, every other one in the Hessian determinant is negative. Or

$$\frac{\partial^2 P}{\partial I^2}<0 \quad \text{and} \quad \begin{vmatrix} \frac{\partial^2 P}{\partial I^2} & \frac{\partial^2 P}{\partial I\,\partial CD} \\ \frac{\partial^2 P}{\partial CD\,\partial I} & \frac{\partial^2 P}{\partial CD^2} \end{vmatrix} > 0.$$

[3]In actuality, any individual bank J can change -- in particular, reduce its CD liabilities -- relatively slowly by allowing them to mature or by holding back the issuance of new CD's. However, this problem is of less significance.

Differentiating the FOC, it is written:

$$\frac{\partial^2 P}{\partial I^2} = -\ (r-r_s) \cdot \frac{\partial\Phi(-\overline{FR})}{\partial(-\overline{FR})} \cdot \frac{\partial(-\overline{FR})}{\partial I}$$

or

$$\frac{\partial^2 P}{\partial I^2} = -\ (r-r_s) \cdot \frac{\partial\Phi(-\overline{FR})}{\partial(-\overline{FR})} < 0$$

or

$$r-r_s > 0 \qquad r > r_s.$$

$$\begin{vmatrix} \frac{\partial^2 P}{\partial I^2} & \frac{\partial^2 P}{\partial I \partial CD} \\ \frac{\partial^2 P}{\partial CD \partial I} & \frac{\partial^2 P}{\partial CD^2} \end{vmatrix} = \left[-\ (r-r_s)\ \frac{\partial\Phi(-\overline{FR})}{\partial(-\overline{FR})} \right] \cdot$$

$$\left[\frac{\partial e}{\partial CD} - \frac{\partial^2 e}{\partial CD^2} \cdot CD - \frac{\partial e}{\partial CD} + (r-r_s)(1-\gamma) \cdot \frac{\partial\Phi(-\overline{FR})}{\partial(-\overline{FR})} \cdot (1-\gamma) \right]$$

$$-\left[-\ (r-r_s)\ \frac{\partial\Phi}{\partial(-\overline{FR})} \cdot (1-\gamma) \right]^2$$

$$= 2\ \frac{\partial e}{\partial CD} + \frac{\partial^2 e}{\partial CD^2}\ CD - 2(i-r_s)(1-\gamma)^2 > 0.$$

Given $\overline{FR}_J^*$ and CD_J^* from equations (49) and (51), the optimum value for investment from equation (47) has resulted as:

$$I_J^* = \overline{D}_J(1-\delta) + \overline{T}_J(1-\tau) - \overline{CL}_J + \Phi_J^{-1}(\frac{i-r_s}{r-r_s})$$

$$+q_J(i,r_{CD},CL_J) \cdot (1-\gamma) + CA_J. \tag{52}$$

Assuming all banks face the same interest rates and the same reserve requirement ratios, the equations for the aggregate banking system can be written:

$$\overline{FR}^* = \sum_{J=1}^{n_0} \overline{FR}_J^* = - \psi(\frac{i-r_s}{r-r_s}) \tag{53}$$

$$\frac{\partial \overline{FR}^*}{\partial i} < 0, \quad \text{and} \quad \frac{\partial \overline{FR}^*}{\partial r} > 0.$$

Similarly, for the banking system as a whole[4] it can be stated that:

$$CD^* = \sum_{J=1}^{n_0} CD_J^* = \sum_{J=1}^{n_0} q_J(i,r_{CD},CL_J)$$

---

[4]In general, large banks issue CD's and small banks buy CD's. The banks get no reduction in the required reserves against their CD liabilities as they buy CD's issued by other banks. Therefore, the large banks do not buy the CD's issued by other banks. On the other hand, small banks do not issue CD's because there is no market for them. In this study, CD's are considered only as the liabilities of the banking system. CD liabilities are relatively larger in volume and play a relatively more important role in the adjustment process of the banking system than CD's do as assets.

or

$$CD^{*} = Q(i, r_{CD}, CL),^{5,6} \tag{54}$$

where it is assumed

$$CL = \sum_{J=1}^{n_0} CL_J$$

and Q function exhibits the same properties as q_J function.

---

[5]In the banking system, at any point in time a large amount of issued CD's are matured and a large number of new CD's are issued. Therefore, in the banking system CD's can be assumed to be relatively flexible as any of the explanatory variables, i, r_{CD}, and CL change. Hence, the problem stated in Footnote 3 becomes less significant from the point of view of the banking system as a whole.

[6]Alternatively, equation (54) can be derived by using the linear approximation for the average subjective cost of issuing CD by bank J. It can be stated:

$$e_J(CD_J, CL_J) = A_{0J} + A_{1J}CD_J + A_{2J}CL_J$$

where

$$\frac{\partial e_J}{\partial CD_J} = A_{1J} > 0 \quad \text{and} \quad \frac{\partial e_J}{\partial CL_J} = A_{2J} < 0.$$

By substituting the above relationship in equation (50), the result is:

$$-A_{0J} - A_{1J}CD_J - A_{2J}CL_J - A_{1J}CD_J = r_{CD} - (1-\gamma)i.$$

Thus:

$$CD_J^{*} = \frac{-A_{0J}}{2A_{1J}} + \frac{1-\gamma}{2A_{1J}} i - \frac{1}{2A_{1J}} r_{CD} - \frac{A_{2J}}{2A_{1J}} CL_J.$$

Then, in aggregate for the banking system, it is written as:

$$CD^{*} = \left[- \sum_{J=1}^{n_0} \frac{A_{0J}}{2A_{1J}} \right] + \left[(1-\gamma) \sum_{J=1}^{n_0} \frac{1}{2A_{1J}} \right] i$$

(Footnote continued on following page).

The balance sheet and identities for the banking system as a whole can be written as follows.

Balance Sheet of the Banking System

Assets				Liabilities		
Reserves Total			R	Demand Deposits		D
Required		RR		Federal Government	D_g	
against demand deposit	δD			All Others	D_p	
against time deposit	τT			Time Deposits		T
against CD	γCD			CD		CD
Surplus		S		Borrowing		B^*
excess reserves	ER			from FRB's	B	
loans in FFM	FFL			Federal Fund borrowings	FFB	
Commercial loans			CL	Miscellaneous liabilities and Capital		MA+CA
Other Investments			I			
Miscellaneous Assets			MA			

(Footnote continued from preceding page).

$$- \left[\sum_{J=1}^{n_0} \frac{1}{2A_{IJ}} \right] r_{CD} + \left[- \sum_{J=1}^{n_0} \frac{A_{2J}}{2A_{1J}} \right] CL.$$

Thus, $$CD^* = B_0+(1-\gamma)B_1 i-B_1 r_{CD}+B_2 CL \tag{55}$$

where $$\frac{\partial CD^*}{\partial i} = (1-\gamma)B_1 = (1-\gamma) \sum_{J=1}^{n_0} \frac{1}{2A_{1J}} > 0,$$

(Footnote continued on following page).

$$RR \equiv \delta D + \tau T + \gamma CD, \tag{56}$$

$$FR \equiv D+T-RR-CL-I+CD+CA,$$

or

$$FR \equiv D(1-\delta)+T(1-\tau)+CD(1-\gamma)-CL-I+CA. \tag{57}$$

The aggregate of $\overline{FR}$ from equation (47) would be

$$\overline{FR} = \overline{D}(1-\delta)+\overline{T}(1-\tau)-\overline{CL}-I+CD(1-\gamma)+CA. \tag{58}$$

The optimum level for the demand for investment by the banking system can be found by substituting for the values of $\overline{FR}^*$ and CD^* from equations (53) and (54) in equation (58).

$$I^* = \overline{D}(1-\delta)+\overline{T}(1-\tau)-\overline{CL}+\psi\left(\frac{i-r_s}{r-r_s}\right)+(1-\gamma)Q(i,r_{CD},CL)+CA\ . \tag{59}$$

Assume there exist nondecreasing marginal subjective costs of issuing CD in the banking system,

or

$$\frac{\partial(MSC)}{\partial CD} \geq 0$$

where

$$MSC = \frac{\partial e}{\partial CD}\ .$$

In other words, the higher the level of outstanding CD, the higher the subjective cost is for issuing additional CD. Moreover, assume

(Footnote continued from preceding page).

$$\frac{\partial CD}{\partial r_{CD}} = -B_1 = -\sum_{J=1}^{n_0} \frac{1}{2A_{1J}} < 0,$$

$$\frac{\partial CD^*}{\partial CL} = B_2 = -\sum_{J=1}^{n_0} \frac{A_{2J}}{2A_{1J}} > 0.$$

$$\frac{\partial(MSC)}{\partial CL} \leq 0,$$

which means that the higher the demand for commercial loans, the lower the subjective cost is for issuing additional CD. Equation (50) in the aggregate level can be rewritten as equation M;

$$M = e(CD,CL) + \frac{\partial e}{\partial CD}(CD,CL) \cdot CD + r_{CD} - (1-\gamma)i = 0. \tag{60}$$

To obtain the comparative static results, CD can be differentiated with respect to i, r_{CD}, and CL. Using implicit function theorem from equation (60) it can be stated that:

$$\frac{\partial CD^*}{\partial i} = -\frac{\partial M/\partial i}{\partial M/\partial CD^*} = -\frac{-(1-\gamma)}{\frac{\partial e}{\partial CD} + \frac{\partial e}{\partial CD} + CD\,\frac{\partial^2 e}{\partial CD^2}} > 0,$$

$$\frac{\partial CD^*}{\partial r_{CD}} = -\frac{\partial M/\partial r_{CD}}{\partial M/\partial CD^*} = \frac{1}{\frac{\partial e}{\partial CD} + \frac{\partial e}{\partial CD} + CD \cdot \frac{\partial^2 e}{\partial CD^2}} < 0,$$

$$\frac{\partial CD^*}{\partial CL} = -\frac{\partial M/\partial CL}{\partial M/\partial CD^*} = -\frac{\frac{\partial e}{\partial CL} + CD \cdot \frac{\partial^2 e}{\partial CL \cdot \partial CD}}{\frac{\partial e}{\partial CD} + \frac{\partial e}{\partial CD} + CD \cdot \frac{\partial^2 e}{\partial CD^2}} > 0.$$

The optimum investment from equation (59) can be differentiated with respect to i, r_{CD}, r, and CL

$$\frac{\partial I^*}{\partial i} = \frac{\partial \psi}{\partial i} + \frac{\partial Q}{\partial i}(1-\gamma) > 0,$$

$$\frac{\partial I^*}{\partial r} = \frac{\partial \psi}{\partial r} < 0,$$

$$\frac{\partial I^*}{\partial r_{CD}} = \frac{\partial Q}{\partial r_C} (1-\gamma) < 0,$$

$$\frac{\partial I^*}{\partial CL} = - \frac{\partial \overline{CL}}{\partial CL} + \frac{\partial Q}{\partial CL} (1-\gamma), \text{ which has an ambiguous sign.}$$

With the same assumption about the expectations of the variables, as stated in Part II, equation (59) can be rewritten as:

$$\begin{aligned} I^* = {} & (1-\delta) \left[n_D(D-D_{-1})+D_{-1}\right] + (1-\tau) \left[n_T(T-T_{-1})+T_{-1}\right] \\ & - \left[n_{CL}(CL-CL_{-1})+CL_{-1}\right] - \left[n_F(\overline{FR}^*-FR_{-1})+FR_{-1}\right] \\ & + (1-\gamma) \left[(CD^*-CD_{-1})+CD_{-1}\right] + \left[(CA-CA_{-1})+CA_{-1}\right] \end{aligned}$$

or

$$\begin{aligned} & I^* - \left[(1-\delta)D_{-1}+(1-\tau)T_{-1}-CL_{-1}-FR_{-1}+(1-\gamma)CD_{-1}+CA_{-1}\right] \\ & = n_D(1-\delta)\Delta D+n_T(1-\tau)\Delta T-n_{CL}\Delta CL-n_F(\overline{FR}^*-FR_{-1}) \\ & \quad + (1-\gamma)CD^* - (1-\gamma)CD_{-1}+\Delta CA. \end{aligned}$$

Thus

$$\begin{aligned} I^* = {} & n_D(1-\delta)\Delta D+n_T(1-\tau)\Delta T-n_{CL}\Delta CL-n_F(\overline{FR}^*-FR_{-1}) \\ & + (1-\gamma)CD^*-(1-\gamma)CD_{-1}+ CA. \end{aligned} \tag{61}$$

From the balance sheet of the banking system, it is known

$$R+CL+I \equiv D+T+CD+B^{*}+CA$$

or

$$UR+CL+I \equiv D+T+CD+CA,$$

so

$$\Delta D = \Delta UR-\Delta T+\Delta CL-\Delta CD^{*}-\Delta CA+\Delta I^{*}, \tag{62}$$

where: $\Delta I = \Delta I^{*}$ and $\Delta CD = \Delta CD^{*}$.

By substituting for the values of ΔI^{*} and CD^{*} in equation (62) the supply of deposit for the banking system is derived

$$\Delta D = \Delta UR-\Delta T+\Delta CL-\Delta CA-CD^{*}+CD_{-1}+n_D(1-\delta)\Delta D$$

$$+n_T(1-\tau)\Delta T-n_{CL}\Delta CL-n_F(\overline{FR}^{*}-FR_{-1})+(1-\gamma)CD^{*}$$

$$-(1-\gamma)CD_{-1}+\Delta CA,$$

or

$$\Delta D = \frac{1}{1-n_D(1-\delta)}\Big[\Delta UR-(1-n_T+\tau n_T)\Delta T+(1-n_{CL})\Delta CL$$

$$-n_F(\overline{FR}^{*}-FR_{-1}) - \gamma CD^{*}+\gamma CD_{-1}\Big] . \tag{63}$$

Briefly considering the CD market, it can be written

$$CD^D = CD^D(r_{CD}, i, Y)$$

where CD^D = demand for CD by nonbank-public; it is reasonable to assume that the demand for CD, like any other demand for financial securities or certificates, is a function of its own rate and the interest rate on close substitute securities and income level or GNP. Moreover, it is reasonable to write

$$\frac{\partial CD^D}{\partial r_{CD}} > 0, \qquad \frac{\partial CD^D}{\partial i} < 0, \qquad \frac{\partial CD^D}{\partial Y} > 0.$$

Equation (54) can be restated as the desired supply of CD by the banking system

$$CD^S = CD^S(i, r_{CD}, CL)$$

where

$$\frac{\partial CD^S}{\partial r_{CD}} < 0, \qquad \frac{\partial CD^S}{\partial i} > 0, \qquad \frac{\partial CD^S}{\partial CL} > 0.$$

For the CD market equilibrium, it can be stated:

$$CD^S = CD^D$$

or

$$CD^S(r_{CD}, i, CL) - CD^D(r_{CD}, i, Y) = 0.$$

Thus,

$$r_{CD} = R_{CD}(i, Y, CL), \tag{64}$$

where

$$\frac{\partial r_{CD}}{\partial i} = - \frac{\dfrac{\partial CD^S}{\partial i} - \dfrac{\partial CD^D}{\partial i}}{\dfrac{\partial CD^S}{\partial r_{CD}} - \dfrac{\partial CD^D}{\partial r_{CD}}} > 0,$$

$$\frac{\partial r_{CD}}{\partial Y} = - \frac{- \dfrac{\partial CD^D}{\partial Y}}{\dfrac{\partial CD^S}{\partial r_{CD}} - \dfrac{\partial CD^D}{\partial r_{CD}}} < 0,$$

$$\frac{\partial r_{CD}}{\partial CL} = - \frac{\frac{\partial CD^S}{\partial CL}}{\frac{\partial CD^S}{\partial r_{CD}} - \frac{\partial CD^D}{\partial r_{CD}}} > 0.$$

Substituting r_{CD} from equation (64) into equation (54), the result is:

$$CD^* = Q[i, CL, R_{CD}(i, Y, CL)]$$

or

$$CD^* = Q^a(i, CL, Y) \tag{65}$$

where

$$\frac{\partial CD^*}{\partial CL} = \frac{\partial Q}{\partial CL} + \frac{\partial Q}{\partial r_{CD}} \cdot \frac{\partial r_{CD}}{\partial CL} > 0,$$ [7]

$$\frac{\partial CD^*}{\partial Y} = \frac{\partial Q}{\partial r_{CD}} \cdot \frac{\partial r_{CD}}{\partial Y} > 0,$$

$$\frac{\partial CD^*}{\partial i} = \frac{\partial Q}{\partial i} + \frac{\partial Q}{\partial r_{CD}} \cdot \frac{\partial r_{CD}}{\partial i}$$

which has an ambiguous sign and depends on relative elasticities of the supply of and the demand for CD with respect to i and r_{CD}.

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$$\frac{\partial CD^*}{\partial CL} = \frac{\partial CD^S}{\partial CL} + \frac{\partial CD^S}{\partial r_{CD}} \left[- \frac{\frac{\partial CD^S}{\partial CL}}{\frac{\partial CD^S}{\partial r_{CD}} - \frac{\partial CD^D}{\partial r_{CD}}} \right] =$$

$$\frac{\frac{\partial CD^S}{\partial CL} \cdot \frac{\partial CD^S}{\partial r_{CD}} - \frac{\partial CD^S}{\partial CL} \cdot \frac{\partial CD^D}{\partial r_{CD}} - \frac{\partial CD^S}{\partial r_{CD}} \cdot \frac{\partial CD^S}{\partial CL}}{\frac{\partial CD^S}{\partial r_{CD}} - \frac{\partial CD^D}{\partial r_{CD}}} = \frac{-\frac{\partial CD^S}{\partial CL} \cdot \frac{\partial CD^D}{\partial r_{CD}}}{\frac{\partial CD^S}{\partial r_{CD}} - \frac{\partial CD^D}{\partial r_{CD}}} > 0.$$

From the FFM, the equations (13) and (14) in Part II can be restated:

$$\overline{FR}^* = F(i,r)$$

$$r = R(d,FR).$$

Thus,

$$\overline{FR}^* = F[i,R(d,FR)]$$

or

$$\overline{FR}^* = h^a[i,d,FR] \tag{66}$$

where

$$\frac{\partial\overline{FR}^*}{\partial i} < 0, \qquad \frac{\partial\overline{FR}^*}{\partial d} > 0, \qquad \frac{\partial\overline{FR}^*}{\partial FR} < 0.$$

Finally, it is known from the balance sheet for the banking system that

$$FR \equiv UR-\delta D-\tau T-\gamma CD. \tag{67}$$

Then, using equations (66), (65), (63), and the identity (67), the structural equations for the system are restated as a system:

$$\begin{cases} 1) \quad \overline{FR}^* = h^a(i,d,FR) \\ 2) \quad CD^* = Q^a(i,Y,CL) \\ 3) \quad \Delta D = \dfrac{1}{1-n_D(1-\delta)}\,[\Delta UR+(1-n_{CL})\Delta CL-(1-n_T+\tau n_T)\Delta T \\ \qquad\qquad - n_F(\overline{FR}^*-FR_{-1})-\gamma CD^*+\gamma CD_{-1}] \\ 4) \quad FR \equiv UR-\delta D-\tau T-\gamma CD^*. \end{cases}$$

Assuming banks adjust to their desired level of free reserves, the actual level of free reserves in the banking system is derived from the above structural equations:

$$FR = \frac{\delta n_F}{1-n_D(1-\delta)} h^a(i,d,FR) + \left[1- \frac{\delta n_F}{1-n_D(1-\delta)}\right] FR_{-1}$$

$$+ \left[1- \frac{\delta}{1-n_D(1-\delta)}\right] \Delta UR - \left[\tau - \frac{\delta(1-n_T+\tau n_T)}{1-n_D(1-\delta)}\right] \Delta T$$

$$- \frac{\delta(1-n_{CL})}{1-n_D(1-\delta)} \Delta CL - \left[1- \frac{\delta}{1-n_D(1-\delta)}\right] \cdot \gamma \cdot Q^a(i,Y,CL)$$

$$+ \left[1- \frac{\delta}{1-n_D(1-\delta)}\right] \cdot \gamma \cdot CD_{-1}. \tag{68}$$

Define:

$$\Gamma = \frac{\delta n_F}{1-n_D(1-\delta)} = \frac{\delta n_F}{(1-n_D)+\delta n_D} .$$

Then, it is seen

$$0 < \Gamma < 1.$$

On the other hand, n_D is small realistically; therefore,

$$0 < \Gamma/n_F = \frac{\delta}{(1-n_D)+\delta n_D} < 1.$$

Equation (67) is rewritten as:

$$FR = \Gamma h^a(i,d,FR) + (1-\Gamma)FR_{-1} + (1-\Gamma/n_F)\Delta UR - \left[\tau-\Gamma/n_F(1-n_T+\tau n_T)\right] \Delta T$$

$$-\Gamma/n_F(1-n_{CL})\Delta CL - (1-\Gamma/n_F)\gamma\cdot Q^a(i,Y,CL)+(1-\Gamma/n_F)\gamma CD\text{-}1, \tag{69}$$

where

$$\frac{\partial FR}{\partial i} = - \frac{\Gamma \frac{\partial \overline{FR}^*}{\partial i} - (1-\Gamma/n_F)\gamma \cdot \frac{\partial CD^*}{\partial i}}{-1+\Gamma \cdot \frac{\partial \overline{FR}^*}{\partial FR}}$$

which has an ambiguous sign.

$$\frac{\partial FR}{\partial d} = - \frac{\Gamma \frac{\partial \overline{FR}^*}{\partial d}}{-1+\Gamma \frac{\partial \overline{FR}^*}{\partial FR}} > 0$$

$$\frac{\partial FR}{\partial Y} = - \frac{-(1-\Gamma/n_F)\gamma \cdot \frac{\partial CD^*}{\partial Y}}{-1+\Gamma \cdot \frac{\partial \overline{FR}^*}{\partial FR}} > 0$$

$$\frac{\partial FR}{\partial CL} = - \frac{-\Gamma/n_F(1-n_{CL}) - (1-\Gamma/n_F)\gamma \frac{\partial CD^*}{\partial CL}}{-1+\Gamma \cdot \frac{\partial \overline{FR}^*}{\partial FR}} < 0.$$

Equation (69) can be statistically estimated and the predicted value of FR can be substituted in the identity of

$$D^S \equiv 1/\delta\, UR - \tau/\delta\; T - \gamma/\delta CD - 1/\delta \hat{FR}$$

to predict the level of the deposit supply for the banking system.

Inclusion of the Eurodollars and certificates of deposit transactions in Model I (Model II under lagged reserve accounting)

Eurodollar (EURO) deposits are those denominated in dollars and accepted by the banks outside the United States. All EURO deposits have some fixed term which ranges from overnight to five years. Most EURO transactions are in the range of six months and under. The banks borrow and lend in dollar denominations among themselves and nonbank-corporations in the EURO market. The overnight EURO loans and borrowings are the alternative sources to the FFM for loans and borrowings for U.S. commercial banks. The longer term (thirty to ninety days) EURO funds are the alternative sources of the funds from issuing CD's and selling Treasury bills. Therefore, the rates on overnight EURO borrowings closely track FFR and the rates on longer term EURO borrowings closely track the money market rates such as r_{CD} and i. Under the period which is considered in empirical works in Chapter IV, the Fed imposed under Regulations M and D a reserve requirement against EURO borrowings. Currently, Regulations M and D require banks to hold reserve requirements against net EURO borrowings[1] over a twenty-eight day averaging period. The technicality of the way that reserve requirements against EURO borrowings are calculated, and the ways that EURO transactions are cleared, make it possible for the banks to arbitrage between overnight EURO market and FFM. Similarly, the banks arbitrage between the longer term EURO market and CD, or the Treasury bill market.

While fixed term, fixed rate loans in the EURO market are uncommon, in this part, the short-term (three-months) EURO borrowings with fixed

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[1]Borrowings minus loans.

interest rates are included in Model I under LRA. Short-term EURO borrowings compete with issuing CD's or selling Treasury bills. Therefore, in this part, it is assumed that banks try to optimize their investment, CD, and EURO borrowing portfolios. Based on these optimum values, the banks adjust their FFM transactions and borrowings from the FRB's. Therefore, the banks choose the decision variables I, CD, and EURO borrowings to maximize the expected level of their profit. The variables $\overline{D}$, $\overline{T}$ and $\overline{CL}$ are known. The reserve ratios and CA are constant with respect to banking behavioral relationships. The balance sheet of a representative bank is:

<u>Balance Sheet of Representative Bank</u>

Assets				Liabilities		
Reserves total			R_t	Demand Deposits		D_t
Required		RR_t		Federal Government	D_{g_t}	
against demand deposits	δD_{t-2}			All Others	D_{p_t}	
against time deposits	τT_{t-2}			Time Deposits		T_t
against CDs	γCD_{t-2}			CD		CD_t
against EURO borrowings	ωU_{t-2}			EURO Borrowings		U_t
				Borrowings		B_t^*
Surplus		S_t		from FRB's	B_t	
excess reserves	ER_t			from FFM	FFB_t	
loans in FFM	FFL_t			Miscellaneous liabilities		MA_t+
Commercial loans			CL_t	and Capital		CA_t
Other Investments			I_t			
Miscellaneous Assets			MA_t			

where ω = Required reserve ratio against U;
U_t = EURO borrowings at time t.

The definition of the other variables in the above balance sheet are the same as seen in Model II under CRA.

The following identities for the representative bank can be written

$$RR_t \equiv \delta D_{t-2}+\tau T_{t-2}+\gamma CD_{t-2}+\omega U_{t-2} \tag{70}$$

$$FR_t \equiv D_t-T_t-RR_t-CL_t-I_t+CD_t+U_t+CA_t. \tag{71}$$

With the same behavioral assumptions as seen in Part II, the outcome of FR_t depends upon the realization of the variables D_t, T_t, and CL_t for any chosen value for I_t, CD_t, and U_t by the profit maximization procedure. For the representative bank, it can be written

$$\begin{aligned} FR_t &= \overline{FR}_t + \tilde{X}_t \\ \overline{FR}_t &= \overline{D}_t + \overline{T}_t - \overline{CL}_t - I_t + CD_t + U_t - RR_t + CA_t, \\ \tilde{X}_t &= \tilde{X}_{D_t} + \tilde{X}_{T_t} + \tilde{X}_{CL_t}. \end{aligned} \tag{72}$$

Given RR_t based on the past values of variables and constant CA_t, the optimization of the variables I_t, CD_t, and U_t would optimize $\overline{FR}_t$.

Assuming the representative individual bank J is a small component of the banking system from equation (72) it is shown that:

$$\frac{\partial \overline{FR}_{J_t}}{\partial I_{J_t}} = -1, \quad \frac{\partial \overline{FR}_{J_t}}{\partial CD_{J_t}} = +1, \quad \frac{\partial \overline{FR}_{J_t}}{\partial U_{J_t}} = +1.$$

Bank J's expected profit can be stated as:

$$P_{J_t} = K_{J_t} + r_{CL}\overline{CL}_{J_t} + iI_{J_t} - \left[r_{CD} + e_{J_t}\right]CD_{J_t} - \left[r_U + \varepsilon_{J_t}\right]U_{J_t}$$

$$+ r_s\overline{FR}_{J_t} + (r - r_s)\int_{-\infty}^{-\overline{FR}_{J_t}} (\overline{FR}_{J_t} + \tilde{X}_{J_t})dG_{J_t}(\tilde{X}_{J_t}) \qquad (73)$$

where

r_U = Thirty to ninety days EURO borrowing rate;

ε_{J_t} = The average subjective cost of borrowing from the EURO market.

The subjective cost for borrowing from the EURO market, for the representative individual bank J, can be explained by the following variables.

1) Outstanding level of EURO borrowing - The higher the U, the higher the subjective cost is for borrowing an additional unit from the EURO market.
2) Outstanding level of CD - Similarly the higher the outstanding level of CD issued by bank J, the higher the subjective cost is for borrowing from the EURO market. The larger CD liabilities or U liabilities puts the bank's soundness more under question.
3) Amount of commercial loans - Banks use funds from sources like CD issuance and EURO market borrowing to finance their commercial loans (see Stigum (37)). Thus, the higher the demand for commercial loans, the lower the subjective cost is for the borrowing from the EURO market.

Other less significant variables such as London interbank offered rates on EURO (LIBOR) and economic or political states of foreign countries can also be considered as explanatory variables for explaining the subjective cost of borrowing EURO's.[2] Only three variables, CD, U, and CL, would be considered as explanatory variables of ε_{J_t} in this study. Moreover, the subjective cost of issuing CD would also depend upon the level of EURO borrowings in this part. The higher the level of EURO borrowing, the higher the subjective cost for issuing additional CD is expected. As a result, it can be stated that

$$e_{J_t} = e_{J_t}(CD_{J_t}, U_{J_t}, CL_{J_t})$$

where

$$\frac{\partial e_{J_t}}{\partial CD_{J_t}} > 0, \quad \frac{\partial e_{J_t}}{\partial U_{J_t}} > 0, \quad \text{and} \quad \frac{\partial e_{J_t}}{\partial CL_{J_t}} < 0.$$

And,

$$\varepsilon_{J_t} = \varepsilon_{J_t}(U_{J_t}, CD_{J_t}, CL_{J_t})$$

where

$$\frac{\partial \varepsilon_{J_t}}{\partial U_{J_t}} > 0, \quad \frac{\partial \varepsilon_{J_t}}{\partial CD_{J_t}} > 0, \quad \text{and} \quad \frac{\partial \varepsilon_{J_t}}{\partial CL_{J_t}} < 0.$$

To maximize the expected profit, equation (75) is differentiated with respect to decision variables I_{J_t}, CD_{J_t}, and U_{J_t}.

$$\frac{\partial P_{J_t}}{\partial I_{J_t}} = i+(r-r_s)\,\frac{\partial(-\overline{FR}_{J_t})}{\partial I_{J_t}}\;\frac{\partial \int_{-\infty}^{-\overline{FR}_{J_t}} (\tilde{X}_{J_t}+\overline{FR}_{J_t})\,d\Phi_{J_t}(\tilde{X}_{J_t})}{\partial(-\overline{FR}_{J_t})}$$

$$+\; r_s\,\frac{\partial \overline{FR}_{J_t}}{\partial I_{J_t}}$$

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[2]For the purpose of short-run study, these variables are considered to be constant.

$$\frac{\partial P_{J_t}}{\partial CD_{J_t}} = -\left[r_{CD}+e_{J_t}\right] - \left[\frac{\partial e_{J_t}}{\partial CD_{J_t}}\right] CD_{J_t} - \left[\frac{\partial \varepsilon_{J_t}}{\partial CD_{J_t}}\right] \cdot U_{J_t}$$

$$+ (r-r_s)\,\frac{\partial(-\overline{FR}_{J_t})}{\partial CD_{J_t}} \cdot \frac{\partial \int_{-\infty}^{-\overline{FR}_{J_t}} (\tilde{X}_{J_t}+\overline{FR}_{J_t})\,d\Phi_{J_t}(\tilde{X}_{J_t})}{\partial(-\overline{FR}_{J_t})}$$

$$+ r_s\,\frac{\partial \overline{FR}_{J_t}}{\partial CD_{J_t}}$$

$$\frac{\partial P_{J_t}}{\partial U_{J_t}} = -\left[\frac{\partial e_{J_t}}{\partial U_{J_t}}\right] \cdot CD_{J_t} - \left[r_U + \varepsilon_{J_t}\right] - \left[\frac{\partial \varepsilon_{J_t}}{\partial U_{J_t}}\right] \cdot U_{J_t}$$

$$+(r-r_s)\,\frac{\partial(-\overline{FR}_{J_t})}{\partial U_{J_t}} \quad \frac{\partial \int_{-\infty}^{-\overline{FR}_{J_t}} (\tilde{X}_{J_t}+\overline{FR}_{J_t})\,d\Phi_{J_t}(\tilde{X}_{J_t})}{\partial(-\overline{FR}_{J_t})}$$

$$+ r_s\,\frac{\partial \overline{FR}_{J_t}}{\partial U_{J_t}} \quad .$$

Therefore, FOC can be stated as:

$$i+(r-r_s)\left[-\Phi(-\overline{FR}_{J_t})\right] - r_s = 0$$

$$-(r_{CD}+e_{J_t}) - (\frac{\partial e_{J_t}}{\partial CD_{J_t}}) \cdot CD_{J_t} - (\frac{\partial \varepsilon_{J_t}}{\partial CD_{J_t}}) \cdot U_{J_t} - (r-r_s) \cdot$$

$$\left[-\Phi_{J_t}(-\overline{FR}_{J_t})\right] + r_s = 0$$

$$- (\frac{\partial e_{J_t}}{\partial U_{J_t}}) \cdot CD_{J_t} - (r_U + \varepsilon_{J_t}) - (\frac{\partial \varepsilon_{J_t}}{\partial U_{J_t}}) \cdot U_{J_t} - (r-r_s) \cdot$$

$$\left[- \quad \Phi_{J_t}(-\overline{FR}_{J_t})\right] + r_s = 0.$$

Thus:

$$\Phi(-\overline{FR}_{J_t}) = \frac{i-r_s}{r-r_s}$$

or

$$\overline{FR}^*_{J_t} = -\Phi_{J_t}^{-1}(\frac{i-r_s}{r-r_s}) \ . \qquad (74)$$

It can also be written:

$$-e_{J_t}(CD_{J_t},U_{J_t},CL_{J_t}) - \frac{\partial e_{J_t}}{\partial CD_{J_t}}(CD_{J_t},U_{J_t},CL_{J_t}) \cdot CD_{J_t}$$

$$- \frac{\partial \varepsilon_{J_t}}{\partial CD_{J_t}} (CD_{J_t},U_{J_t},CL_{J_t}) \cdot U_{J_t} = r_{CD}-i \qquad (75)$$

and

$$- \varepsilon_{J_t}(CD_{J_t}, U_{J_t}, CL_{J_t}) - \frac{\partial e_{J_t}}{\partial U_{J_t}}(CD_{J_t}, U_{J_t}, CL_{J_t}) \cdot CD_{J_t}$$

$$- \frac{\partial \varepsilon_{J_t}}{\partial U_{J_t}}(CD_{J_t}, U_{J_t}, CL_{J_t}) \cdot U_{J_t} = r_U - i. \tag{76}$$

Then, the optimum value of CD_{J_t} and U_{J_t} can be stated as:

$$CD^*_{J_t} = o1_{J_t}(i, r_{CD}, r_U, CL_{J_t}) \tag{77}$$

$$U^*_{J_t} = o2_{J_t}(i, r_{CD}, r_U, CL_{J_t})^{3} \tag{78}$$

The SOC for the profit maximization are stated as:

$$\frac{\partial^2 p}{\partial_I^2} < 0, \quad \begin{vmatrix} \frac{\partial^2 p}{\partial I^2} & \frac{\partial^2 p}{\partial I \partial CD} \\ \frac{\partial^2 p}{\partial I \partial CD} & \frac{\partial^2 p}{\partial CD^2} \end{vmatrix} > 0, \text{ and}$$

$$\begin{vmatrix} \frac{\partial^2 p}{\partial_I^2} & \frac{\partial^2 p}{\partial I \partial CD} & \frac{\partial^2 p}{\partial I \partial U} \\ \frac{\partial^2 p}{\partial I \partial CD} & \frac{\partial^2 p}{\partial CD^2} & \frac{\partial^2 p}{\partial CD \partial U} \\ \frac{\partial^2 p}{\partial I \partial U} & \frac{\partial^2 p}{\partial CD \partial U} & \frac{\partial^2 p}{\partial_U^2} \end{vmatrix} < 0.$$

[3]The usual practice in term-EURO transactions is to price them on a rollover basis (i.e., every month the EURO borrowings are repriced). Hence, U_{J_t} can be assumed to be relatively flexible as any of the explanatory variables i, r_{CD}, r_U, CL_{J_t} change.

Given $\overline{FR}^*_{J_t}$, $CD^*_{J_t}$, and $U^*_{J_t}$ from equations (74), (77), and (78), the optimum value for investment from equation (72) for bank J has resulted as:

$$I^*_{J_t} = \overline{D}_{J_t} + \overline{T}_{J_t} - \overline{CL}_{J_t} + \phi_{J_t}^{-1} \left(\frac{i-r_s}{r-r_s}\right)$$

$$+ o1_{J_t}(i, r_{CD}, r_U, CL_{J_t}) + o2_{J_t}(i, r_{CD}, r_U, CL_{Jt})$$

$$- RR_{J_t} + CA_{J_t}. \tag{79}$$

Assuming all banks face the same interest rates and the same reserves requirements ratios, for the banking system in aggregate it can be written

$$\overline{FR}^*_t = \sum_{J=1}^{n_0} \overline{FR}^*_{J_t} = -\psi\left(\frac{i-r_s}{r-r_s}\right) \tag{80}$$

$$\frac{\partial \overline{FR}^*}{\partial i} < 0, \quad \text{and} \quad \frac{\partial \overline{FR}^*}{\partial r} > 0$$

$$CD^*_t = \sum_{J=1}^{n_0} CD^*_{J_t} = \sum_{J=1}^{n_0} o1_{J_t}(i, r_{CD}, r_U, CL_{J_t})$$

or

$$CD^*_t = O1_t(i, r_{CD}, r_U, CL_t) \tag{81}$$

$$U^*_t = \sum_{J=1}^{n_0} U^*_{J_t} = \sum_{J=1}^{n_0} o2_{J_t}(i, r_{CD}, r_U, CL_{J_t})$$

or

$$U^*_t = O2_t(i, r_{CD}, r_U, CL_t). \tag{82}$$

ψ, 01 and 01 exhibit the same properties, respectively, as Φ^{-1}, o1, and o2.

The balance sheet of the representative bank and the identities of (70) and (71) and equation (72) can be used for the banking system as a whole by redefining the variables as their aggregate levels. The optimum level for the demand for investment by the banking system can be written as:

$$I_t^* = \overline{D}_t+\overline{T}_t-\overline{CL}_t+\psi_t\left(\frac{i-r_s}{r-r_s}\right) +01_t(i,r_{CD},r_U,CL_t)$$

$$+\ 02_t(i,r_{CD},r_U,CL_t) - RR_t+CA_t. \tag{83}$$

Taking into account the values of expectations from Part II, the above equation can be rewritten as:

$$I_t^* = \left[n_D(D_t-D_{t-1})+D_{t-1}\right] + \left[n_T(T_t-T_{t-1}) + T_{t-1}\right]$$

$$- \left[n_{CL}(CL_t-CL_{t-1})+CL_{t-1}\right] - \left[n_F(\overline{FR}_t^*-FR_{t-1})+FR_{t-1}\right]$$

$$+ \left[(CD_t^*-CD_{t-1})+CD_{t-1}\right] + \left[(U_t^*-U_{t-1})+U_{t-1}\right]$$

$$- \left[(RR_t-RR_{t-1})+RR_{t-1}\right] + \left[(CA_t-CA_{t-1})+CA_{t-1}\right]$$

or

$$\Delta I_t^* = n_D\Delta D_t+n_T\Delta T_t-n_{CL}\Delta CL_t-n_F(\overline{FR}_t^*-FR_{t-1})+\Delta CD_t^*$$

$$+\Delta U_t^*-\Delta RR_t+\Delta CA_t. \tag{84}$$

From the balance sheet of the banking system, it is known:

$$R_t+CA_t+I_t \equiv D_t+T_t+CD_t+U_t+B_t^*+CA_t,$$

or

$$UR_t+CL_t+I_t \equiv D_t+T_t+CD_t+U_t+CA_t,$$

so

$$\Delta D_t = \Delta UR_t-\Delta T_t+\Delta CL_t-\Delta CD_t^*-\Delta U_t^*-\Delta CA_t+\Delta I_t^*. \tag{85}$$

The supply of deposit for the banking system is derived as:

$$\Delta D_t = \Delta UR_t-\Delta T_t+\Delta CL_t-\Delta CD_t^*-\Delta U_t^*+\Delta CA_t+n_D\Delta D_t$$

$$+n_T\Delta T_t-N_{CL}\Delta CL_t-n_F(\overline{FR}_t^*-FR_{t-1})+\Delta CD_t^*+\Delta U_t^*-\Delta RR_t-\Delta CA_t$$

or

$$\Delta D_t = \frac{1}{1-n_D}\Big[\Delta UR_t-(1-n_T)\Delta T_t+(1-n_{CL})\Delta CL_t$$

$$-n_F(\overline{FR}_t^*-FR_{t-1})-\Delta RR_t\Big]. \tag{86}$$

As observed above, the current desired level of changes in CD and U has no effect on the change in the deposit supply level. Given fixed UR_t and the stable path of currency in the nonbank-public's hand, the changes in current CD_t and U_t can affect the level of deposit supply due to the change in required reserves. Under the LRA system, it is known

$$RR_t = \delta D_{t-2}+\tau T_{t-2}+\gamma CD_{t-2}+\omega U_{t-2}.$$

Therefore, the changes in CD_t and U_t can only affect D_t after two weeks. Under the assumptions of the model, we find no direct link between D_t and CD_t or U_t.

Under LRA from the FFM, equation (39) from Part II can be restated:

$$\overline{FR}_t^* = h(i_t, d_t, UR_t)$$

$$\frac{\partial \overline{FR}_t^*}{\partial i} < 0, \quad \frac{\partial \overline{FR}_t^*}{\partial d} > 0, \text{ and } \frac{\partial \overline{FR}_t^*}{\partial UR_t} < 0.$$

Hence, the reduced form on the change in the level of deposit supply is derived as:

$$\Delta D_t = \frac{-n_F}{1-n_D} h(i_t, d_t, UR_t) + \frac{1}{1-n_D} UR_t - \frac{1-n_F}{1-n_D} UR_{t-1} - \frac{1-n_T}{1-n_D} \Delta T_t$$

$$+ \frac{1-n_{CL}}{1-n_D} \Delta CL_t - \frac{1}{1-n_D} RR_t + \frac{1-n_F}{1-n_D} RR_{t-1} \qquad (87)$$

or

$$\Delta D_t = \frac{-n_F}{1-n_D} h(i_t, d_t, UR_t) + \frac{1}{1-n_D} UR_t - \frac{1-n_F}{1-n_D} UR_{t-1} - \frac{1-n_T}{1-n_D} \Delta T_t$$

$$-\frac{\tau}{1-n_D} T_{t-2} + \frac{\tau(1-n_F)}{1-n_D} T_{t-3} + \frac{1-n_{CL}}{1-n_D} \Delta CL_t - \frac{\gamma}{1-n_D} CD_{t-2} + \frac{\gamma(1-n_F)}{1-n_D} CD_{t-3}$$

$$- \frac{\omega}{1-n_D} U_{t-2} + \frac{\omega(1-n_F)}{1-n_D} U_{t-3} - \frac{\delta}{1-n_D} D_{t-2} + \frac{\delta(1-n_F)}{1-n_D} D_{t-3}. \qquad (88)$$

Define

$$\Gamma 1 = \frac{n_F}{1-n_D} .$$

Expectation on D_t by the banks is relatively imperfect, so n_D is very small, so $0 < \Gamma 1 < 1$. It is also true that

$$\frac{\Gamma 1}{n_F} = \frac{1}{1-n_D} \geq 1.$$

Equations (87) and (88) then can be written as

$$\Delta D_t = -\Gamma 1 h(i_t, d_t, UR_t) + \frac{\Gamma 1}{n_F} UR_t - \frac{\Gamma 1}{n_F}(1-n_F)UR_{t-1} - \frac{\Gamma 1}{n_F}(1-n_T)\Delta T_t$$

$$+ \frac{\Gamma 1}{n_F}(1-n_{CL})\Delta CL_t - \frac{\Gamma 1}{n_F} RR_t + \frac{\Gamma 1}{n_F}(1-n_F)RR_{t-1} \tag{89}$$

and

$$\Delta D_t = -\Gamma 1 h(i_t, d_t, UR_t) + \frac{\Gamma 1}{n_F} UR_t - \frac{\Gamma 1}{n_F}(1-n_F)UR_{t-1} - \frac{\Gamma 1}{n_F}(1-n_T)\Delta T_t$$

$$- \frac{\Gamma 1}{n_F}\tau \cdot T_{t-2} + \frac{\Gamma 1}{n_F}\tau \cdot (1-n_F)T_{t-3} + \frac{\Gamma 1}{n_F}(1-n_{CL})\Delta CL_t - \frac{\Gamma 1}{n_F} \cdot \gamma CD_{t-2}$$

$$+ \frac{\Gamma 1}{n_F} \cdot \gamma (1-n_F)CD_{t-3} - \frac{\Gamma 1}{n_F} \cdot \omega \cdot U_{t-2} + \frac{\Gamma 1}{n_F}\omega(1-n_F)U_{t-3}$$

$$- \frac{\Gamma 1}{n_F}\delta D_{t-2} + \frac{\Gamma 1}{n_F}\delta(1-n_F)D_{t-3}. \tag{90}$$

The following specifications about the signs of the coefficients of the above equations can be made:

$$-1 < \text{Coeff of } UR_{t-1} = -\frac{\Gamma 1}{n_F}(1-n_F) < 0$$

$$-1 < \text{Coeff of } \Delta T_t = -\frac{\Gamma 1}{n_F}(1-n_T) < 0$$

$$0 < \text{Coeff of } \Delta CL_t = \frac{\Gamma 1}{n_F}(1-n_{CL}) < 1$$

$$\text{Coeff of } RR_t = -\frac{\Gamma 1}{n_F} \leq -1$$

$$0 < \text{Coeff of } RR_{t-1} = \frac{\Gamma 1}{n_F}(1-n_F) < 1$$

$$-1 < \text{Coeff of } T_{t-2} = -\frac{\Gamma 1}{n_F}\tau < 0$$

$$0 < \text{Coeff of } T_{t-3} = \frac{\Gamma 1}{n_F}\tau(1-n_F) < 1$$

$$-1 < \text{Coeff of } CD_{t-2} = -\frac{\Gamma 1}{n_F}\cdot\gamma < 0$$

$$0 < \text{Coeff of } CD_{t-3} = +\frac{\Gamma 1}{n_F}\gamma(1-n_F) < 1$$

$$-1 < \text{Coeff of } U_{t-2} = -\frac{\Gamma 1}{n_F}\cdot\omega < 0$$

$$0 < \text{Coeff of } U_{t-3} = +\frac{\Gamma 1}{n_F}\omega\cdot(1-n_F) < 1$$

$$-1 < \text{Coeff of } D_{t-2} = -\frac{\Gamma 1}{n_F}\delta < 0$$

$$0 < \text{Coeff of } D_{t-3} = +\frac{\Gamma 1}{n_F}\delta(1-n_F) < 1.$$

With the same linear approximation as seen in Part II, it can be written:

$$\Delta D_t = -\Gamma 1a_0 - \Gamma 1a_1 i_t - \Gamma 1a_2 d_t + (\Gamma 1n_F - \Gamma 1a_3)UR_t - \frac{\Gamma 1}{n_F}(1-n_F)UR_{t-1}$$
$$-\frac{\Gamma 1}{n_F}(1-n_T)\Delta T_t + \frac{\Gamma 1}{n_F}(1-n_{CL})\Delta CL_t - \frac{\Gamma 1}{n_F}RR_t + \frac{\Gamma 1}{n_F}(1-n_F)RR_{t-1} \tag{91}$$

or

$$\Delta D_t = -\Gamma 1a_0 - \Gamma 1a_1 i_t - \Gamma 1a_2 d_t + \left(\frac{\Gamma 1}{n_F} - \Gamma 1a_3\right) UR_t - \frac{\Gamma 1}{n_F}(1-n_F) UR_{t-1}$$

$$- \frac{\Gamma 1}{n_F}(1-n_T)\Delta T_t - \frac{\Gamma 1}{n_F}\tau T_{t-2} + \frac{\Gamma 1}{n_F}\tau(1-n_F) T_{t-3} + \frac{\Gamma 1}{n_F}(1-n_{CL})\Delta CL_t$$

$$- \frac{\Gamma 1}{n_F}\gamma CD_{t-2} + \frac{\Gamma 1}{n_F}\gamma(1-n_F) CD_{t-3} - \frac{\Gamma 1}{n_F}\cdot\omega\cdot U_{t-2} + \frac{\Gamma 1}{n_F}\omega(1-n_F) U_{t-3}$$

$$- \frac{\Gamma 1}{n_F}\delta D_{t-2} + \frac{\Gamma 1}{n_F}\delta(1-n_F) D_{t-3} \qquad (92)$$

where

Coeff of $i_t = -\Gamma 1a_1 > 0$

Coeff of $d_t = -\Gamma 1a_2 < 0$

Coeff of $UR_t = + \frac{\Gamma 1}{n_F} - \Gamma 1a_3 > 0.$

Part IV. Model III

Model II under Federal fund rate instrument with inclusion of repurchase agreement transactions using lagged reserve accounting

The main tools available to the Fed in implementing its policies are: open-market operations (OMO), required reserve ratios, and discount rates. By the open market operation, we mean that FRB open market committee (FOMC) sells or buys the USGS's,[1] hence, changes banks' reserves to achieve its target. Since eighty percent of the value of the Fed's assets consisted of USGS, the OMO has come to be the most important tool for policy implementation by the Fed. Obviously, the first task of the FOMC is to pursue a macro-economic target variable like full employment, price stability, or a stable exchange rate for the dollar. The FOMC can try to achieve any of the above desirable targets via channels of money supply or the interest rate change. Having chosen its target, the FOMC has to decide which of the above channels to use. During the decade of the 1960s, the FOMC gradually shifted its focus from interest rate strategy to money supply strategy by attempting to keep the growth of money supply on a targeted path. At the same time, the kind of instructions given by FOMC for implementing OMO showed a narrower band for controlling FFR's variations. Therefore, it seems after the mid-sixties, FOMC has controlled the FFR exogenously as a short-run instrumental variable to control the money supply path. In contrast to that, before the mid-sixties

[1]In other words, FOMC demonotizes or monotizes some portion of national debt.

reserve aggregates (i.e., UR) as short-run instrumental variables seemed to play the dominant role.[2]

Theoretically, to explain the relationships among the monetary variables more realistically for the time period undertaken by the empirical work of this study, in this part the FFR is considered as the short-run instrumental variable used by FOMC exogeneously to control money supply. In Parts II and III, UR is taken as exogenous and as a short-run instrumental variable. The problem in Parts II and III is then, if UR is truly endogeneous, it would be affected by random shocks created by demand side variations by the nonbank-public. Thus, the models in Parts II and III are mis-specified and have incorrectly channeled the nonbank-public's random shocks to affect the deposit supply. In this part, it is assumed that the individual bank optimizes its investment portfolios and thus, CD and U liabilities. Then, based on these optimum values, the bank adjusts its FFM transactions, borrowings from FRB, excess reserves and/or RP transactions.

A repurchase agreement (RP) is an arrangement between a commercial bank and a corporate first to sell and then to repurchase a security. RP is a secured loan with a fixed rate of return and maturity. Generally, the maturity date on RP's is overnight although term RP's (thirty-days or more) are also transacted. A reverse repurchase agreement (RV) is an arrangement first to buy and then to resell a security. RV is the opposite of RP. The most common type of securities used in RP and RV market are the Treasury bill and other USGS's. If USGS's are used as collateral, banks are not

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[2]The policy of controlling an optimal combination of a reserve aggregate and the FFR as a short-run instrumental variable to control money supply, might explain the behavior of FOMC at its best. This might be a subject of future study.

required to hold any reserve requirement against RP or RV transactions. Overnight RP or RV transactions are the alternatives for FFM transactions. The interest rate on overnight RP or RV keeps track of FFR and is usually lower than that.

In this part, Model II under LRA from Part III is modified as follows:

1) FFR is used as the short-run instrumental variable in contrast to reserve aggregates (i.e., UR_t).
2) The overnight RP and RV transactions among the banks with USGS as collateral are added as two alternatives to the other short-run portfolios. It is assumed that banks withhold RP and RV at the same time. The balance sheet for the banking system can be shown as:

Balance Sheet of Banking System

Assets				Liabilities		
Reserve total			R_t	Demand deposit		D_t
Required reserves		RR_t		by government	D_g	
against demand deposit	δD_{t-2}			by others	D_p	
against time deposit	τT_{t-2}			Time deposit		T_t
against CD	γCD_{t-2}			CD		CD_t
against EURO borrowings	ωU_{t-2}			EURO borrowings		U_t
Surplus		S_t		Borrowings		B_t^*
Excess reserves	ER_t			from FRB	B_t	
Federal Fund loans	FFL_t			from FFM	FFB_t	
Reverse repurchase agreement	RV_t			repurchase agreement	RP_t	
Commercial loans			CL_t	Miscellaneous liabilities		MA+
Other earning assets (investment)			I_t	Capital		CA
Miscellaneous assets			MA_t			

where: $S_t \equiv ER_t+FFL_t+RV_t$ and $B_t^* \equiv B_t+FFB_t+RP_t$.

The other identities in the balance sheet are the same as the identities used for Model II under LRA in Part III.

With the same behavioral assumptions as seen in Part III, the optimum values CD^*, U^*, I^*, and $\overline{FR}^*$ for the banking system are found. Equation (84) is restated as:

$$\Delta I_t^* = n_D \Delta D_t + n_T \Delta T_t - n_{CL} \Delta CL_t - n_F \overline{FR}_t^* - n_F FR_{t-1} + \Delta CD_t^* + \Delta U_t^* - \Delta RR_t + \Delta CA_t. \quad (93)$$

From the balance sheet of the banking system, we derive:

$$\Delta D_t \equiv \Delta FR_t - \Delta T_t + \Delta CL_t - \Delta CD_t - \Delta U_t + \Delta RR_t + \Delta I_t + \Delta CA_t$$

where

$$\Delta FR_t = \Delta S_t - \Delta B_t^*.$$

By substituting for ΔI^* from equation (93) in the above identity, the result is:

$$\Delta D_t = \frac{1}{1-n_D}\left[FR_t - n_F \overline{FR}_t^* - (1-n_T)\Delta T_t + (1-n_{CL})\Delta CL_t - (1-n_F)FR_{t-1}\right]. \quad (94)$$

Under the system of FFR, as an instrumental variable both UR_t and FR_t are endogenously determined in the system. From Appendix I, FFM under FFR instrument it is obtained:

$$FR_t = FR(r_F, d, r_R).$$

It is also restated:

$$\overline{FR}_t^* = \Omega 1(i, r_F, r_R, d).$$

Thus, equation (94) can be rewritten as:

$$\Delta D_t = \frac{1}{1-n_D} FR(r_F,d,r_R) - \frac{n_F}{1-n_D} \Omega I(i,r_F,r_R,d)$$

$$- \frac{1-n_T}{1-n_D} \Delta T_t + \frac{1-n_{CL}}{1-n_D} \Delta CL_t - \frac{1-n_F}{1-n_D} FR_{t-1}. \tag{95}$$

The expected sign of the coefficients for the variables can be written as follows:

$$\text{Coeff. for } i \approx \frac{\partial[\Delta D_t]}{\partial i} = - \frac{n_F}{1-n_D} \cdot \frac{\partial \overline{FR}_t^*}{\partial i} \geq 0$$

$$\text{Coeff. for } r_F \approx \frac{\partial[\Delta D_t]}{\partial r_F} = \frac{1}{1-n_D} \frac{\partial FR_t}{\partial r_F} - \frac{n_F}{1-n_D} \cdot \frac{\partial \overline{FR}_t^*}{\partial r_F} \leq 0$$

$$\text{Coeff. for } d \approx \frac{\partial[\Delta D_t]}{\partial d} = \frac{1}{1-n_D} \cdot \frac{\partial FR_t}{\partial d} - \frac{n_F}{1-n_D} \cdot \frac{\partial \overline{FR}_t^*}{\partial d}$$, it has an ambiguous sign which depends on which of the actual or desired free reserves are more responsive to change in the discount rate.

$$\text{Coeff. for } r_R \approx \frac{\partial[\Delta D_t]}{\partial r_R} = \frac{1}{1-n_D} \cdot \frac{\partial FR_t}{\partial r_R} - \frac{n_F}{1-n_D} \frac{\partial \overline{FR}_t^*}{\partial r_R}$$, it has an ambiguous sign.

$$-1 \leq \text{Coeff. for } \Delta T_t = - \frac{1-n_T}{1-n_D} \leq 0$$

assuming n_D is actually very small

$$0 \leq \text{Coeff for } \Delta CL_t = + \frac{1-n_{CL}}{1-n_D} \leq 1$$

$$-1 < \text{Coeff. for } FR_{t-1} = \frac{1-n_F}{1-n_D} < 0$$

If FR_t and $\overline{FR}_t^*$ are both linearly approximated, then it can be written:

$$FR_t = \gamma_0+\gamma_1 r_F+\gamma_2 d+\gamma_3 r_R,$$

$$\overline{FR}_t^* = \lambda_0+\lambda_1 r_F+\lambda_2 d+\lambda_3 r_R+\lambda_4 i.$$

Hence:

$$\Delta D_t = \frac{\gamma_0-n_F\lambda_0}{1-n_D} - \frac{\lambda_4 n_F}{1-n_D} i_t + \frac{\gamma_1-n_F\lambda_1}{1-n_D} r_{F_t}$$

$$+ \frac{\gamma_2-n_F\lambda_2}{1-n_D} d_t + \frac{\gamma_3-n_F\lambda_3}{1-n_D} r_{R_t} - \frac{1-n_T}{1-n_D} \Delta T_t$$

$$+ \frac{1-n_{CL}}{1-n_D} \Delta CL_t - \frac{1-n_F}{1-n_D} FR_{t-1}. \tag{96}$$

It is known that

$$FR_{t-1} \equiv UR_{t-1}-RR_{t-1}$$

$$RR_{t-1} \equiv \delta D_{t-3}+ \tau T_{t-3} + \gamma CD_{t-3} + \omega U_{t-3}.$$

Then, equation (96) is rewritten as:

$$\Delta D_t = \frac{\gamma_0-n_F\lambda_0}{1-n_D} - \frac{\lambda_4 n_F}{1-n_D} i_t + \frac{\gamma_1-n_F\lambda_1}{1-n_D} r_{F_t} + \frac{\gamma_2-n_F\lambda_2}{1-n_D} d_t$$

$$+ \frac{\gamma_3-n_F\lambda_3}{1-n_D} r_{R_t} - \frac{1-n_T}{1-n_D} \Delta T_t + \frac{1-n_{CL}}{1-n_D} \Delta CL_t$$

$$- \frac{1-n_F}{1-n_D} UR_{t-1} + \frac{1-n_F}{1-n_D} RR_{t-1}. \tag{97}$$

Or,

$$\Delta D_t = \frac{\gamma_0 - n_F\lambda_0}{1-n_D} - \frac{\lambda_4 n_F}{1-n_D} i_t + \frac{\gamma_1 - n_F\lambda_1}{1-n_D} r_{F_t} + \frac{\gamma_2 - n_F\lambda_2}{1-n_D} d_t$$

$$+ \frac{\gamma_3 - n_F\lambda_3}{1-n_D} r_{R_t} - \frac{1-n_T}{1-n_D} \Delta T_t + \frac{1-n_{CL}}{1-n_D} \Delta CL_t$$

$$- \frac{1-n_F}{1-n_D} UR_{t-1} + \frac{1-n_F}{1-n_D} \delta D_{t-3} + \frac{1-n_F}{1-n_D} \tau T_{t-3}$$

$$+ \frac{1-n_F}{1-n_D} \gamma CD_{t-3} + \frac{1-n_F}{1-n_D} \omega U_{t-3}. \tag{98}$$

In equations (97) and (98) we can specify the coefficients of other variables as follows:

$$-1 < \text{Coeff. for } UR_{t-1} = - \frac{1-n_F}{1-n_D} < 0$$

$$0 < \text{Coeff. for } RR_{t-1} = \frac{1-n_F}{1-n_D} < 1$$

$$0 < \text{Coeff. for } D_{t-3} = \frac{1-n_F}{1-n_D} \cdot \delta < 1$$

$$0 < \text{Coeff. for } T_{t-3} = \frac{1-n_F}{1-n_D} \cdot \tau < 1$$

$$0 < \text{Coeff. for } CD_{t-3} = \frac{1-n_F}{1-n_D} \gamma < 1$$

$$0 < \text{Coeff. for } U_{t-3} = \frac{1-n_F}{1-n_D} \omega < 1.$$

CHAPTER IV. EMPIRICAL RESULTS

Part I. Statistical Problems

Seasonality

Any time series Y_t can be described by the following components:

T = Trend: The long-run growth or decline in the time series data.

C = Cycles: The fluctuations in the time series with a duration of 2-10 years (occasionally more).

I = Irregular fluctuations: The erratic movements in the time series data with no regular or recognizable pattern.

S = Seasonal variations: The variations in the time series with a periodic pattern and a duration of one year and less.

Seasonal variations have been modelled by two types of representations:

1) Additive seasonal components - The magnitudes of seasonal "swings" of the time series are independent of the average level of the time series as determined by the trend

$$Y_t = T_t+C_t+S_t+I_t.$$

2) Multiplicative seasonal components - The magnitudes of seasonal "swings" of the time series are proportional to the average level of the time series as determined by trend. As the average level of the time series changes, so do the magnitudes of the seasonal "swings".

$$Y_t = T_t \cdot C_t \cdot S_t \cdot I_t \ .$$

In the real world, the variations of all time series are only approximated by all of the above variations. When fitting the deposit supply functions resulting from models in Chapter III, distortions due to the seasonal component may unfold in two ways:

1) when seasonally unadjusted data are used in the estimations, and
2) when deseasonalized data are used, but different procedures are utilized for adjustment of the different series.

The assumption, for the legitimate use of seasonally unadjusted data in the estimating procedures, is that the seasonal component in the dependent variable is explained by the seasonal components in the independent variables. If the variations of the dependent variables are not all explained by those of the independent variables, dummy variables or trigonometric functions can be used as additional explanatory variables. The problem with this approach is that it is not known whether the seasonal variations of the dependent variable are fully explained by those of the independent variables or other dummy or trigonometric variables. Hence, the false inferences about the significance of the explanatory variables could result. The irregular term would not be normally distributed and would include the seasonal component. Thus, the estimation of coefficients of explanatory variables would be biased.

Deseasonalized observations are found by taking out the seasonal factor from the original time series data which eliminates the seasonal effects. This can be accomplished in both frequency and time domains. In the frequency domain, a certain transformation of the autocovariance function of the time series is utilized. This transformation is called Fourier transformation or spectral density function.

$$f(\omega) = \frac{1}{2\Pi} \sum_{h=-\infty}^{\infty} \rho(h)\rho^{-i\ h}$$

where

$\rho(h)$ = autocovariance function of the time series

$f(\omega)$ = periodic function of period 2π.

Therefore, the resultant index is represented in frequencies of various trigonometric functions. The contributions by each of the periodic components decomposed by spectral analysis, to the total variance of the time series would be determined. Thus, the spectral density would identify the seasonal (periodic) component. The spectral density of deseasonalized data should be approximately equal to the power of the white noise around each of the seasonal frequencies.

For the purpose of deseasonalization in the time domain, if the time series has additive seasonal variations in general, then the linear regression of the time series on dummy variables or trigonometric functions gives better results in terms of smaller seasonal biases.[1] If the time series has multiplicative variations using the decomposition method[2] or regressing the time series on trigonometric functions, a better performance would then result. The additive regression model can be written in a linear form

$$Y_t = T_t+S_t+I_t \quad [3]$$

---

[1]Seasonal bias is measured by the sum of the squared differences between the observed and predicted values of both historical and future data.

[2]In a more complex approach X-11 method.

[3]Cyclical variations are assumed to be absent.

where

$$S_t = \sum_{J=1}^{n} \beta_{S_J} D_{j,t}$$

and

$$D_{j,t} = \text{dummy variable } j$$

$$T_t = \text{a linear function for trend variable.}$$

The seasonal "swings" of a time series with additive seasonal components are independent of the trend function which is described by the above model. In a time series with multiplicative seasonal components, the seasonal swings are related to trend of the time series which the above model is not able to explain. In multiplicative decomposition methods, the moving average of the time series is computed using exactly one observation from each season. That would eliminate seasonal and irregular components and the averaged series would consist of $T_t X C_t$. Then, it follows

$$S_t X I_t = \frac{Y_t}{T_t \times C_t} \quad .$$

By computing the seasonal factor, the seasonal component of the time series is estimated and deseasonalized data is found by dividing the observations on Y_t by the seasonal factor (see Bowerman and O'Connell (5)). Hence, this method does not efficiently explain a time series with additive trend, cycle, seasonal components.

As mentioned, the same deseasonalizing procedure should be utilized for adjusting the different series. Otherwise, the seasonal bias in the overall model may be intensified. The effects of the seasonal bias on the estimation results may be difficult to recognize. Furthermore, it would be difficult to compare the empirical results. If all the time series data in a model are deseasonalized by the same procedure, the procedure should be an optimum one in such a manner that the minimum seasonal bias remains in the model.

For the purpose of deseasonalization, all weekly time series data are regressed on quarterly dummy variables in this study. The reasons for this procedure are that, at present, there is not a good seasonal adjustment procedure for weekly data of the type employed in this study. Secondly, the plots for observations and spectral density functions for the key variables indicate spikes reflecting strong quarterly seasonality. Thus, quarterly dummies are chosen as a second best, but hopefully reasonable way to capture the important seasonal movements in the series. Therefore, the variable Y_t is regressed on matrix S_t and the residual of the regression, e_t, is taken as the seasonally adjusted variable, where

$$Y_t = \hat{\beta}S_t + e_t,$$

and

$$S_t = \begin{bmatrix} S_0 & S_1 & S_2 & S_3 \end{bmatrix},$$

and

$$e_t = Y_t - \hat{\beta}S_t.$$

S_0 is a column of one's and S_1, S_2, and S_3 are the quarterly seasonal dummies; or

$$S_t = \left.\begin{array}{l}
\left.\begin{array}{rrrr} 1 & 1 & 0 & 0 \\ \vdots & & & \\ 1 & 1 & 0 & 0 \end{array}\right\} \text{13 weeks in the first quarter} \\
\left.\begin{array}{rrrr} 1 & 0 & 1 & 0 \\ \vdots & & & \\ 1 & 0 & 1 & 0 \end{array}\right\} \text{13 weeks in the second quarter} \\
\left.\begin{array}{rrrr} 1 & 0 & 0 & 1 \\ \vdots & & & \\ 1 & 0 & 0 & 1 \end{array}\right\} \text{13 weeks in the third quarter} \\
\left.\begin{array}{rrrr} 1 & -1 & -1 & -1 \\ \vdots & & & \\ 1 & -1 & -1 & -1 \end{array}\right\} \text{13 weeks in the fourth quarter} \\
\begin{array}{rrrr} \vdots & \vdots & \vdots & \vdots \end{array}
\end{array}\right\} \text{first year}$$

The values of columns in S_t matrix in the first year are repeated for the second, and subsequent years for the period of the study. Some of the variables in the study might have no important seasonal component, but the seasonal adjustment on them is justified by the assumption that the seasonal dummies are nearly orthogonal. Therefore, in the regression equations of these variables, the seasonal dummies have little or no explanatory power and the residuals are almost equal to the original variables. In fact, the above adjustment procedure might not be optimal, but should be adequate for the purpose of this study. All but two of the variables in this study were originally not seasonally adjusted. The two variables are GNP = Y and GNP price deflator = P. Both were officially seasonally adjusted. There were no available data on unadjusted series for these two variables. As mentioned previously, this problem might create distortions due to filtering and recognizing the seasonal noise. But, for two reasons, this problem is not very significant. First, both of the above variables do not come into the picture in a single equation approach for deposit supply estimation. By utilizing only the demand deposit equation in a simultaneous equation approach, the above variables, Y and P, can have an effect on the deposit supply estimation. Secondly, it is highly possible that the official seasonal adjustment method used for the variables, Y and P, is a quarterly seasonal adjustment because the original data on Y and P were obtained as quarterly data which are interpolated to weekly data. Therefore, a close similarity in official seasonal adjustment on Y and P and the seasonal adjustment method used in this study on other variables is likely.

Serial correlation

One of the most common assumptions about the disturbance terms in a linear regression equation is that they are independent of one another or the covariance of them is zero. In the following model:

$$Y = \beta X + U,$$

the above assumption implies

$$E(UU') = \sigma^2 I_n$$

$$E(\mu_t \mu_{t+s}) = 0 \quad \text{for } S \neq 0$$

where U = n x 1 matrix of disturbance terms (μ_t's).

The above assumption is not always fulfilled, especially when, as in this study, the observations consist of economic time series. It is often found that the disturbance terms are serially correlated. This problem is known as "autocorrelation." Incorrect specification of a model is one cause of the problem in the model. The disturbance terms would reflect, in part, the effect of omitted relevant variables in an equation. Therefore, in general, any pervasive serial correlation in such variables would create a source for the serial correlation in the disturbance terms. Measurement error in the "explained" variable is another source of autocorrelation in a regression equation.

There exist many different processes in which the disturbance terms can be correlated. The simplest possible type and most common process is a first-order autoregressive process. The following relationship then holds for the disturbance terms:

$$U_t = \rho U_{t-1} + \varepsilon_t$$

where

$$E(\varepsilon_t) = 0$$

$$E(\varepsilon_t, \varepsilon_{t+s}) = \sigma_\varepsilon^2 \quad \text{for } s = 0$$
$$= 0 \quad \text{for } s \neq 0.$$

ε_t is a normally and independently distributed random variable with zero mean and constant variance ($\sigma^2\varepsilon$) and zero covariance. ρ is called the first order autocorrelation coefficient. The above process is called autoregressive because the current value of U is regressed or expressed as a function of the past values of U. It is first order because only one time lag for U is used as an explanatory variable in the above process.

If a model includes significant serially correlated disturbance terms, the use of the ordinary least-squares (OLS) method for the estimation would result in: (1) a nonvalid test of significance for the coefficients of the explanatory variables and for the performance of the model in general, though the estimates for the coefficients remain unbiased; (2) underestimation of variance of the disturbance terms and of the coefficient; and (3) inefficient estimation and prediction power for the model. However, the OLS estimators remain consistent estimators.

Assuming a known first-order autoregressive process and known value for ρ, the use of the generalized least-squares (GLS) method would generate the best linear unbiased estimator for the coefficients of the explanatory variables. Applying GLS method would boil down to the following simple approach: first the matrices Y, X, and U are transformed to the new variables Y^*, X^*, and U^*, where: $Y^* = TY$, $X^* = TX$, and $U^* = TU$, and matrix T is approximated by:

$$T \approx \begin{bmatrix} -\rho & 1 & 0 \ldots 0 & 0 & 0 \\ 0 & -\rho & 1 \ldots 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 \ldots -\rho & 1 & 0 \\ 0 & 0 & 0 \ldots 0 & -\rho & 1 \end{bmatrix}$$

In the next step, the original relationship, using the transformed variables, is estimated by OLS method.

In actuality, both the exact process of autocorrelation and the value of ρ are unknown, and these need to be estimated. The first-order autoregressive process for the disturbance terms is considered in Parts II, III, and IV. However, the second-order autocorrelation is also tested for when significant first-order autocorrelation is found. When the disturbance term follows a first-order autoregressive process, several approaches have been mentioned to estimate ρ. The most accurate, flexible and simple procedure is Durbin's two-stage procedure. By this method, in the absence of any lagged dependent variable as an explanatory variable in the equation, the estimated value of ρ is obtained by fitting the following equation

$$Y_t = \rho Y_{t-1} + X_t \beta - X_{t-1} \rho \beta + \varepsilon_t.$$

$\hat{\rho}$ = the estimated coefficient for Y_{t-1}. The above equation is obtained as follows:

$$Y_t = X_t \beta + U_t$$

$$U_t = \rho U_{t-1} + \varepsilon_t$$

therefore,

$$Y_t - X_t \beta = \rho Y_{t-1} - \rho X_{t-1} \beta + \varepsilon_t.$$

The calculated value for $\hat{\rho}$ would then be substituted for ρ in T matrix.

To test the hypothesis that a significant degree of first-order autocorrelation exists in the model, the "d" statistic by Durbin and Watson (D.W. test), in the absence of any lagged dependent variable as explanatory variable, is used most frequently.

$$d = \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

where e_t's are the residual terms obtained from applying OLS in fitting the model. Two upper and lower values for $d(d_U$ and $d_L)$ for any values of n and k are tabulated in most econometrics text books. To test the hypothesis of zero first-order autocorrelation against the alternative of positive first-order autocorrelation, the hypotheses are:

H_0: zero autocorrelation;

H_A: positive autocorrelation;

if $d < d_L$, reject H_0, accept H_A

if $d > d_U$, fail to reject H_0;

if $d_L < d < d_U$, test is inconclusive.

Similarly:

H_0: zero autocorrelation;

H_A: negative autocorrelation;

if $4\text{-}d < d_L$, reject H_0, accept H_A;

if $4\text{-}d > d_U$, fail to reject H_0;

if $d_L < 4\text{-}d < d_U$, test is inconclusive.

When the explanatory variables include one or more lagged dependent variables, there is no problem of using OLS method to estimate the model and the estimates are unbiased and consistent. However, when lagged dependent variables are some of explanatory variables of a model with a serially correlated disturbance term, then OLS estimates would be inconsistent. Lagged dependent variables cannot be considered as predetermined and fixed explanatory variables because the disturbance terms would be correlated with these explanatory variables which violates one of the basic assumptions in OLS estimation process. In this case, the D.W. test is biased toward the value for random disturbances.

Therefore, the following procedures, as suggested by Fair (14) and Fuller (18), are chosen as the procedure in this study, to estimate the single equation and the simultaneous equation models. In a single equation model, one can consider the case of two lagged dependent and one exogenous explanatory variables in the equation:

$$Y_t = X_t\beta + Y_{t-1}\gamma_1 + Y_{t-2}\gamma_2 + U_t$$

$$U_t = \rho U_{t-1} + \varepsilon_t.$$

The following steps are then approached:

1) Y_{t-1} and Y_{t-2} are regressed on X_{t-1} and X_{t-2}. As a result, $\hat{Y}_{t-1}$ and $\hat{Y}_{t-2}$ are found.

2) Y_t is regressed on X_t, $\hat{Y}_{t-1}$, and $\hat{Y}_{t-2}$, then $\hat{U}_t$ is found.

3) $\hat{\rho} = \dfrac{\sum_{t=2}^{n} \hat{\mu}_t \hat{\mu}_{t-1}}{\sum_{t=2}^{n} \hat{\mu}_{t-1}^2}$ is calculated.

4) $Z_t = (Y_t, X_t, Y_{t-1}, \text{ and } Y_{t-2})$ is transformed as

$$Z_t^* = \sqrt{1-\hat{\rho}^2}\; Z_t \quad \text{for } t=1$$

$$= Z_t - \hat{\rho} Z_{t-1} \quad \text{for } t = 2, 3, \ldots, n.$$

5) From Gauss-Newton procedure, Y_t^* is regressed on X_t^*, Y_{t-1}^*, Y_{t-2}^*, and $\hat{U}_{t-1}$ by OLS approach.

6) $\hat{\hat{\rho}} = \hat{\rho}$ + estimated coefficient of $\hat{U}_{t-1}$ in step 5 is calculated.

7) $t_c = \dfrac{\hat{\hat{\rho}}}{\text{standard deviation of the coefficient of } \hat{U}_{t-1} \text{ in step 5}}$ is calculated.

8) t_c has t distribution; then if $|t_c| > |t_{table}|$, reject the null hypothesis of zero autocorrelation; thus, estimators in step 5 should be used. If $|t_c| < |t_{table}|$, fail to reject H_0. Hence, the initial OLS estimators should be used.

In a simultaneous equation model, assuming no identification problem, it can be written

$$Y_t = X_t\beta + V_t\alpha_1 + W_{t-1}\alpha_2 + U_t$$

$$U_t = \rho U_{t-1} + \varepsilon_t$$

where

X_t is a matrix of the exogenous variables;

V_t is a matrix of other endogenous variables in the system;

W_{t-1} is a matrix of one lagged endogenous variable.

The following steps then are approached:

1) V_t and W_{t-1} are regressed on X_t, X_{t-1}, E_t, and E_{t-1}. As a result, $\hat{V}_t$ and $\hat{W}_{t-1}$ are found (E_t is a matrix for other exogenous variables in the system, but not in the equation).

2) Y_t is regressed on X_t, $\hat{V}_t$, and $\hat{W}_{t-1}$, then $\hat{U}_t$ is found.

3) $\hat{\rho} = \dfrac{\sum_{t=2}^{n} \hat{\mu}_t \hat{\mu}_{t-2}}{\sum_{t=2}^{n} \hat{\mu}_{t-1}^2}$ is calculated.

4) $Z_t = (Y_t, X_t, V_t, W_{t-1}, E_t, \text{ and } E_{t-1})$ is transformed as:

$$Z_t^* = \sqrt{1-\hat{\rho}^2}\; Z_t \quad \text{for } t=1$$

$$= Z_t - \hat{\rho} Z_{t-1} \quad \text{for } t = 2, 3, \ldots, n.$$

5) Y_t^* is regressed on X_t^*, V_t^*, W_t^*, and $\hat{U}_{t-1}$ by two-stage-least-squares (2SLS) approach.

6) $\hat{\hat{\rho}} = \hat{\rho}$ + coefficient of $\hat{U}_{t-1}$ in step 5 is calculated.

7) $t_c = \dfrac{\hat{\hat{\rho}}}{\text{SE(coefficient of } U_{t-1} \text{ in step 5)}}$ is calculated.

8) If $|t_c| > |t_{table}|$, the estimators in step 5 should be used and if $|t_c| < |t_{table}|$, the initial 2SLS estimators should be used.

Part II. Deposit Supply Function under Model I Using Lagged Reserve Accounting

Single equation approach

Equation (44), the first-difference of the deposit supply (ΔD) equation from Model I, is estimated in different versions in this part. To modify the results, the term RL (reserve release term) is added to the equations as an explanatory variable. The term RL shows the effect of the changes in required reserve ratios against demand deposits, time deposits, CDs, and EURO borrowings on ΔD. Ordinary least-square (OLS) regression is used for estimation. When the disturbance terms are "significantly" serially correlated, the equations are corrected by using generalized least-square (GLS). The values in parentheses are the calculated t-values for testing the hypothesis of zero coefficients. The multiple correlation coefficient (R^2), the standard error (SE), the first order autocorrelation ($\hat{\rho}$), and the Durbin-Watson statistic (DW) are listed for each estimation.

List of variables

- i = Three months Treasury bill rate
- d = Discount rate
- $r_{S\&T}$ = Composite rate on time and saving deposits
- r_F = Federal fund market interest rate
- UR = Unborrowed reserves
- RR = Required reserves
- FR = Free reserves
- T = Time and savings deposits at member banks

CD = Certificates of deposit

TD^d = The demand component of the money stock

DG = Government demand deposit at commercial banks

FD = Foreign deposits at the Federal Reserve Banks

Float = Float

CL = Commercial loans

Y = Gross national product

P = GNP price deflator

N = Population

U = Eurodollar borrowings

D = Net demand deposits

IBD^m = Intermember bank deposit

$D1^m$ = Total member bank deposits by nonbank-public and by the United States Government

S = The ratio of $\frac{TD^d + DG}{D1^m}$

δ = Required reserve ratio against net demand deposit

$\tau 1$ = Composite required reserve ratio against T and CD

ω = Required reserve ratio against Eurodollar borrowings

RL = Reserve release[1]

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[1] For a detailed description of the above symbols for variables, see Appendix II, Data directory.

For equation (44) the resulting estimations are shown as:

(44-1)
$$\Delta D_t = \underset{(.057)}{6.81} + \underset{(0.86)}{212.59}\, i_t - \underset{(-2.33)}{386.40} d_t + \underset{(5.81)}{1.73}\, UR_t$$
$$- \underset{(-5.42)}{1.61}\, UR_{t-1} - \underset{(-1.93)}{0.19} \Delta T_t - \underset{(-2.12)}{0.14} T_{t-2} + \underset{(2.92)}{0.19} T_{t-3}$$
$$+ \underset{(6.48)}{1.34} \Delta CL_t - \underset{(-2.55)}{0.16} D_{t-2} - \underset{(-0.18)}{0.01} D_{t-3} + \underset{(0.38)}{0.006} RL_t$$

$R^2 = 0.4087$

SE = 4257138

$\hat{\rho} = -0.1846$

DW = 2.3627

(44-2)
$$\Delta D_t = \underset{(0.87)}{215.41} i_t - \underset{(-1.24)}{390.57} d_t + \underset{(5.82)}{1.72} UR_t - \underset{(-5.44)}{1.61} UR_{t-1}$$
$$- \underset{(-1.97)}{0.19} \Delta T_t - \underset{(-2.17)}{0.14} T_{t-2} + \underset{(2.98)}{0.18} T_{t-3} + \underset{(6.52)}{1.35} \Delta CL_t$$
$$- \underset{(-2.54)}{0.16} D_{t-2} - \underset{(-0.22)}{0.014} D_{t-3}$$

$R^2 = 0.4091$

SE = 4233005

$\hat{\rho} = -0.1837$

DW = 2.3608

(44-3)
$$\Delta D_t = \underset{(0.06)}{8.04} - \underset{(-1.40)}{347.54} i_t + \underset{(0.18)}{55.42} d_t + \underset{(6.40)}{2.24} UR_t$$

$$-1.89UR_{t-1} - 0.29\Delta T_t - 5.17(\tau 1T_{t-2})$$
$$(-5.49) \quad (-2.86) \quad (-3.11)$$

$$+ 4.72(\tau 1T_{t-3}) + 1.48\Delta CL_t - 1.57(\delta D_{t-2})$$
$$(2.97) \quad (6.88) \quad (-3.88)$$

$$0.63(\delta D_{t-3}) + 0.63RL_t$$
$$(1.71) \quad (1.72)$$

R^2 = .3729

SE = 4514943

$\hat{\rho}$ = -0.1120

DW = 2.2180

(44-4) $$\Delta D_t = -345.14i_t + 52.45d_t + 2.24UR_t - 1.89UR_{t-1}$$
$$(-1.41) \quad (0.17) \quad (6.47) \quad (-5.54)$$

$$- 0.29\Delta T_t - 5.16(\tau 1T_{t-2}) + 4.71(\tau 1T_{t-3})$$
$$(-2.87) \quad (-3.11) \quad (2.99)$$

$$+ 1.48\Delta CL_t - 1.57(\delta D_{t-2}) + 0.62(\delta D_{t-3})$$
$$(6.89) \quad (-3.92) \quad (1.72)$$

$$+ 0.63RL_t$$
$$(1.73)$$

R^2 = .3736

SE = 4501108

$\hat{\rho}$ = -0.1121

DW = 2.2181

Equation (44-1) shows the estimation of equation (44) from Model I under LRA in Chapter III, Part II, using OLS. The signs of the coefficients are all as expected except the coefficient of D_{t-3}, which is not significantly different from zero at a 5% level of significance. All variables are regressed on intercept and seasonal dummies as explained in Part I, Seasonality section of this chapter. Therefore, as expected, the intercept term has a very small explanatory power by using deseasonalized data in equation (44-1). The estimated coefficient for RL_t is also insignificantly different from zero which can be explained by the facts that the deseasonalized required reserve ratios have almost no variation during the span of the time period of this study. The time series plot of these ratios supports this hypothesis.

The derived composite required reserve ratio against T and CD ($\tau 1$) was the only ratio with a high degree of fluctuations. Due to the lack of data on the required reserve ratio against T alone, and against CD alone, ratio $\tau 1$ is utilized. This ratio may not show the true variation of the required reserve ratios and the term RL, to explain the variations in the dependent variable ΔD_t. The estimated coefficients on variables i_t and d_t are also not significantly different from zero at a 5% level of significance. However, these two variables and variable D_{t-3} are kept for later discussion.

In equation (44-2) the second estimation of equation (44) from Model I, the intercept and RL terms are removed from the explanatory variables. The general performance of the estimation is improved by having smaller mean square error and larger R^2. The estimated coefficients on variables i_t, d_t, D_{t-3} are still not significantly different from zero.

The implicit assumption in equation (44-1) and (44-2) is that the deseasonalized values of the required reserve ratios are constant during the time period of this study. Therefore, the coefficients of variables T_{t-2}, T_{t-3}, D_{t-2}, and D_{t-3} can be considered as constant parameters. As mentioned before, the time series plot of these ratios supports this hypothesis. Equations (44-3) and (44-4) take into account the variations (if any) of these ratios. At first, the equations (44-3) and (44-4) indicate a significant explanatory power for the variables $(\tau 1T_{t-2})$, $(\tau 1T_{t-3})$, and (δD_{t-2}). This suggests that the required reserve ratios have some explanatory power although their variations are small. Secondly, the lagged deposit variables are absent in equations (44-3) and (44-4), therefore, the general performance of the estimations is worse than equations (44-1) and (44-2) by having a larger SE. Discussed at a later time are the signs and magnitudes of the estimated coefficients of variables i and d.

Equation (44-2), which has the smallest SE and largest R^2 is chosen for further modifications. The estimated coefficient for UR has the expected sign and magnitude. It indicates that the increase in the policy variable by one dollar would induce deposits to increase by more than one dollar which can be expected under the LRA system. The estimated coefficient for UR_{t-1} is negative and greater than one. Its sign is as expected. Its magnitude in absolute value is greater than one, which might suggest n_D is larger than expected. In other words, banks have a relatively accurate anticipation of their deposit level. The magnitude of the estimated coefficient of ΔCL_t also supports the above hypothesis. Its sign is as expected. The increase in demand for the commercial loans causes banks to be more aggressive in making investments, thus increasing their deposits. The sign and magnitude of ΔT is as expected. T can be considered as an alternative source of deposit funds. Thus, other things being the same, as ΔT increases, ΔD would decrease. A prior two week increase in deposits causes the current required reserves to rise and the current demand deposit to fall. Conversely, because of a prior three week rise in deposits, the required reserves of one week previous to the current week would rise, which induces D_{t-1} to fall and ΔD to rise. Therefore, the signs of variables T_{t-2}, T_{t-3}, D_{t-2} appear to be as expected. In reality, the required reserve ratio against demand deposits, on the average, is greater than the required reserve ratio against time deposits. So it is expected that the magnitude of the estimated coefficients for D_{t-2} and D_{t-3} is larger than those for T_{t-2} and T_{t-3}, respectively. As seen, these expectations are not fulfilled by the estimated coefficients in equation (44-2). On the other hand, the significant first-order autocorrelation that existed in the estimation might be responsible for some of the above distortions.

Equation (44-2) corrected for the first-order autocorrelation and using GLS procedure, is shown below in equation (44-5).

$$\text{(44-5)} \qquad \Delta D_t = \underset{(1.23)}{265.34 i_t} - \underset{(-1.64)}{444.35 d_t} + \underset{(6.02)}{1.77 UR_t} - \underset{(-5.61)}{1.65 UR_{t-1}}$$

$$- \underset{(-1.38)}{0.13 \Delta T_t} - \underset{(-1.89)}{0.12 T_{t-2}} + \underset{(2.73)}{0.17 T_{t-3}} + \underset{(6.55)}{1.28 \Delta CL_t}$$

$$- 0.21D_{t-2} + 0.03D_{t-3} - 0.0163\varepsilon_{t-1}$$
$$(-3.5) \quad (0.54) \quad (-0.41)$$

$R^2 = 0.4265$

SE = 4091503

$\hat{\rho} = 0.0016$

DW = 1.9887

In equation (44-2) using OLS estimation, $\hat{\hat{\rho}} = -0.1837$ and DW = 2.3608. The DW test could not be utilized because the upper and the lower limits for d statistics[2] are tabulated for no more than 100 observations and 5 variables. The number of observations in this study is 339. Therefore, the significance of the estimation for ρ is tested directly by t-statistic. For this purpose, the residual of OLS estimation (ε_t) is regressed on its lag (ε_{t-1}). The variables (Z_t) are transformed as $Z_t^* = Z_t - \hat{\rho}Z_{t-1}$. where $\hat{\rho}$ is the estimated coefficient for the lagged residual. Then, the transformed value of the dependent variable is regressed on the transformed values of the independent variables and ε_{t-1} using OLS procedure. The better estimator for the first-order autocorrelation is found as: $\hat{\hat{\rho}} = \hat{\rho}$ + Coeff. of ε_{t-1} in the equation using transformed data. Therefore, in equation (44-2), the first-order autocorrelation is estimated as $\hat{\hat{\rho}} = -0.1837 - 0.0163 = -0.2070$. The calculated t-value for $\hat{\hat{\rho}}$ is shown as

$$t = \frac{-0.2070}{0.0574} = -3.6062$$

where $SE(\varepsilon_{t-2})$ = 0.0574 in the equation using transformed data. As seen above, $|t| > 1.96$ which indicates a significant first-order autocorrelation

[2]See Part I, Serial Correlation Section of this chapter.

at a 5% level of significance. Therefore, equation (44-5), the corrected form for autocorrelation is used instead of equation (44-2). As seen, the first-order autocorrelation in equation (44-5) is not significantly different from zero. Therefore, second-order autocorrelation in equation (44-2) is expected to be not significantly different from zero. Equation (44-5) performs better by having a smaller SE and larger R^2. On the other hand, the estimated coefficient for D_{t-3} is not significantly different from zero, but its sign is as expected in equation (44-5). Furthermore, the estimated coefficient for D_{t-2} is now larger than the estimated coefficient for T_{t-2}, as expected.

Simultaneous equation approach

The theoretical work in Chapter III concentrates on the determination of the deposit supply. In the preceding section of this chapter, the theoretical results are tested using the single equation approach. The first interest, in this part, is to estimate the same deposit supply equation, taking into account the effect of the system of relations in which the deposit supply equation is embedded. This aim is accomplished by using a simultaneous equation approach. The determination of demand for deposit is not studied in this paper. The deposit demand equation formulated by Modigliani et al. (30) is used in this part. The second objective of using the simultaneous equation approach is to be able to forecast two endogenous variables in the money market, namely, demand deposits and the interest rate. The system of equations are shown as follows:

1) $\Delta D_t = \beta_1 i_t + \beta_2 d_t + \beta_3 UR_t + \beta_4 UR_{t-1} + \beta_5 \Delta T_t + \beta_6 T_{t-2} + \beta_7 T_{t-3} + \beta_8 \Delta CL_t$

$+ \beta_9 D_{t-2} + \beta_{10} D_{t-3}.$

2) $TD^d_t = \alpha_1 i_t Y_t + \alpha_2 r_{S\&T_t} Y_t + \alpha_3 Y_t + \alpha_4 N_t P_t + \alpha_5 TD^d_{t-1}.$

3) $D1^m_t \equiv D_t - IBD^m_t - Float_t + FD_t.$

4) $TD^d_t \equiv SD1^m_t - DG_t.$

5) $i_t Y_t \equiv i_t \times Y_t.$

6) $S_t D1^m_t \equiv S_t \times D1^m_t.$

7) $\Delta D_t \equiv D_t - D_{t-1}.$

The first equation is the first difference of the net demand deposit supply from model I under LRA or equation (44-2). The second equation is the demand deposit demand equation or the total banking system deposit by the nonbank-public and the United States Government. The identity (3) shows the definitional relationship between the net demand deposit supply by member banks (D_t) and the total member bank deposits by the nonbank-public and by the United States Government (D^m_1). The identity (4) shows the definitional relationship between TD^d_t and $D1^m_t$. The identities (5) and (6) show the relationships between new variables in the second equation and identity (4), namely iY and $SD1^m$, and variables i, Y, S, $D1^m$. Identity

(7) shows the definitional relationship between the change and the level of the net demand deposit. The endogenous variables in the above system are listed as ΔD_t, i_t, TD^d_t, $i_t Y_t$, $D1^m_t$, D_t, $S_t D1^m_t$.

The exogenous variables are shown as

$$d_t,\ UR_t,\ UR_{t-1},\ \Delta T_t,\ T_{t-2},\ T_{t-3},\ D_{t-2},\ D_{t-3},\ r_{S\&T_t} Y_t,\ \Delta CL_t,$$

$$Y_t,\ N_t P_t,\ TD^d_{t-1},\ IBD^m_t,\ Float_t,\ FD_t,\ DG_t,\ S_t,\ D_{t-1},\ \varepsilon_{t-1}.$$

The term ε_{t-1} is included in the system when the system of equations is corrected for autocorrelation. ε_{t-1} is the lagged value of the residual of the deposit supply equation when 2SLS is used for the above system.

Taking into account the above system of equations and identities, equation (44-2) is estimated by using the two-stage-least-square (2SLS) procedure. The result is shown below in equation (44-6).

$$\text{(44-6)} \quad \Delta D_t = \underset{(-1.15)}{-661.21 i_t} + \underset{(.94)}{655.49 d_t} + \underset{(5.01)}{1.58 UR_t} - \underset{(-5.58)}{1.74 UR_{t-1}} - \underset{(-2.47)}{0.27 \Delta T_t}$$

$$- \underset{(-2.22)}{0.15 T_{t-2}} + \underset{(2.96)}{0.19 T_{t-3}} + \underset{(6.51)}{1.51 \Delta CL_t} - \underset{(-2.25)}{0.15 D_{t-2}} + \underset{(0.43)}{0.03 D_{t-3}}$$

$R^2 = 0.4007$

$SE = 4395817$

$\hat{\rho} = -0.1361$

$DW = 2.2630$

Except the estimated coefficients for i_t and d_t, no other significant changes are obtained by using 2SLS procedure. Using the procedure explained in Serial Correlation Section of Part I of this chapter, equation

(44-6) is corrected for first-order autocorrelation in the context of the simultaneous equation approach. The resultant equation is shown in equation (44-7).

$$\text{(44-7)} \qquad \underset{}{\Delta D_t} = \underset{(0.65)}{351.01 i_t} - \underset{(-0.84)}{546.5 d_t} + \underset{(5.81)}{1.76 UR_t} - \underset{(-5.29)}{1.61 UR_{t-1}} - \underset{(-1.12)}{0.13 \Delta T_t}$$

$$\underset{(-1.97)}{-0.12 T_{t-2}} + \underset{(2.82)}{0.17 T_{t-3}} + \underset{(5.58)}{1.27 \Delta CL_t} \underset{(-3.26)}{-0.21 D_{t-2}}$$

$$+ \underset{(0.28)}{0.02 D_{t-3}} - \underset{(-1.19)}{0.07536 \varepsilon_{t-1}}$$

$R^2 = 0.4209$

$SE = 4101025$

$\hat{\rho} = -0.0064$

$DW = 2.0047$

The better estimator for the first-order autocorrelation coefficient for equation (44-6) is found as:

$$\hat{\hat{\rho}} = -0.1222 - 0.07536 = -0.1976.$$

The calculated t-value for $\hat{\hat{\rho}}$ is shown as:

$$t = \frac{-0.1976}{0.06319} = -3.1266$$

where $SE(\varepsilon_{t-1}) = 0.06319$ in equation (44-7). As seen above, $|t| > 1.96$ which indicates a significant first-order autocorrelation in equation (44-6) at a 5% level of significance. The first-order autocorrelation in equation (44-7) is not significantly different from zero; thus the second-order autocorrelation in equation (44-6) is not expected to be significantly different from zero. Therefore, equation (44-7) in the corrected form, is used for further analysis. The results are the same, except for estimated coefficients of i and d. Equation (44-7) performs better than equation (44-6) by having lower SE and higher R^2.

From the theoretical results of the models in Chapter III, it is expected that: (1) the coefficient of i_t be positive; (2) the coefficient of d_t be negative; and (3) the coefficient of i_t be greater than the coefficient of d_t in absolute value. On the other hand, the FRB's discount rate has been considered to be exogenous throughout the study. But, in reality, the Fed changes the discount rate in part by keeping track of the short-term money market rates. Therefore, a high correlation between i_t and d_t is expected. The time series plot of variables i_t and d_t supports the above hypothesis.[1] Moreover, variable d_t, the explicit discount rate, is not a good measure for the opportunity cost of not borrowing from the FRB's discount window, because a high degree of implicit costs are involved in the borrowing from the FRB. For example, banks become less sound as they borrow from the FRB. Thus, in general, it is expected that d_t has very little significant explanatory power. Finally, another factor which is not undertaken by the models in Chapter III, is the explanatory power of the expectations of the interest rates in explaining the variations of the deposit. Banks usually base their expectations on the current and past values of the variable. Therefore, as i_t increases, banks on one hand are encouraged to invest more, hence increasing their deposits in the process of investment. On the other hand, an increase in i_t might increase

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[1]See Depamphilis (13) and Stigum (37) for more discussion.

their expectations of the future interest rate. Banks would try to economize by lowering their current investments and deposits. So, as seen, the sign of the estimated coefficient for i_t can be both positive and negative.

In equation (44-2), the signs of the estimated coefficients for i_t and d_t are as expected. The magnitude of the coefficient for i_t is less than the magnitude of the coefficient for d_t in absolute value, though both coefficients are not significantly different from zero. This contradicts the theoretical results from Chapter III. The price in the money market, i_t, is most likely subjected to distortion by use of a single equation estimation technique instead of simultaneous equations approach. In equation (44-6), using a simultaneous equation approach, the estimated coefficients of i_t and d_t are still insignificantly different from zero. Moreover, the signs of the coefficients contradict the results of models in Chapter III. The absence of expectations on the interest rates in the model and the other problems mentioned before mgiht explain this contradiction. In addition, the functional form of the demand for demand deposit might be another reason for this behavior. The functional form TD^d is not analyzed in this study. When equation (44-6) is corrected for first-order autocorrelation in equation (44-7), the estimated coefficients for i_t and d_t have correct signs according to the results of the models in Chapter III. The magnitudes are still incorrect. Both variables show little or no explanatory power compared to the large and significant explanatory powers of the policy variable and the lagged exogenous variables.[2]

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[2]The functional form expressed in equation (44) of model I under LRA is used to estimate the level of the deposit supply using no unity

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constraints on the coefficient of the one lagged deposit supply as an explanatory variable. The results are shown below:

(1) Using OLS:

$$D_t = \underset{(0.04)}{4.81} + \underset{(1.37)}{335.07i_t} - \underset{(-1.69)}{523.82d_t} + \underset{(5.71)}{1.66UR_t} - \underset{(-5.12)}{1.5UR_{t-1}}$$

$$- \underset{(-1.12)}{0.11\Delta T_t} - \underset{(-2.36)}{0.15T_{t-2}} + \underset{(3.27)}{0.2T_{t-3}} + \underset{(6.16)}{1.3\Delta CL_t}$$

$$+ \underset{(18.61)}{0.83D_{t-1}} - \underset{(-0.04)}{0.003D_{t-2}} - \underset{(-0.49)}{0.03D_{t-3}}$$

$R^2 = 0.9715$
SE = 4072415
$\hat{\rho} = -0.0320$
DW = 2.0561

2) Using GLS for testing and correcting the first-order autocorrelation;

$$D_t = \underset{(-0.05)}{-5.16} + \underset{(1.34)}{321.63i_t} - \underset{(-1.69)}{508.66d_t} + \underset{(5.81)}{1.71UR_t} - \underset{(-5.25)}{1.55UR_{t-1}}$$

$$- \underset{(-1.1)}{0.11\Delta T_t} - \underset{(-2.18)}{0.14T_{t-2}} + \underset{(3.06)}{0.19T_{t-3}} + \underset{(6.25)}{1.27\Delta CL_t} + \underset{(12.45)}{0.88D_{t-1}}$$

$$- \underset{(-0.66)}{0.06D_{t-2}} - \underset{(-0.3)}{0.02D_{t-3}} - \underset{(-0.3399)}{0.030782\varepsilon_{t-1}} \qquad (SE(\varepsilon_{t-1}) = 0.09055).$$

$R^2 = 0.9747$
SE = 4085873
$\hat{\rho} = 0.0009$
DW = 1.9896

A better estimator for ρ is $\hat{\hat{\rho}} = -0.056947-0.030782 = -0.0877$ and $t = \frac{-0.0877}{0.090550} = -0.9688$; thus $|t| < 1.96$. The first-order autocorrelation in equations using OLS is not significantly different from zero.

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3) Using 2SLS and using the demand equation for the demand deposit utilized in this part:

$$D_t = \underset{(0.045)}{5.27} + \underset{(1.27)}{577.43i_t} - \underset{(-1.31)}{812.7d_t} + \underset{(5.67)}{1.7UR_t} - \underset{(-4.84)}{1.46UR_{t-1}}$$

$$- \underset{(-0.79)}{0.09\Delta T_t} - \underset{(-2.33)}{0.15T_{t-2}} + \underset{(3.26)}{0.21T_{t-3}} + \underset{(5.42)}{1.21\Delta CL_t} + \underset{(17.98)}{0.82D_{t-1}}$$

$$- \underset{(-0.022)}{0.002D_{t-2}} - \underset{(-0.64)}{0.04D_{t-3}}$$

$R^2 = 0.9715$
$\hat{S}E = 4084735$
$\rho = -0.0268$
$DW = 2.0458$

4) Using the procedure explained in Serial Correlation Section of Part I of this chapter, the above equation is tested and corrected for the first-order autocorrelation in the context of simultaneous equation approach.

$$D_t = \underset{(-0.025)}{-2.69} + \underset{(0.96)}{458.66i_t} - \underset{(-1.17)}{673.36d_t} + \underset{(5.8)}{1.75UR_t} - \underset{(-5.15)}{1.56UR_{t-1}}$$

$$- \underset{(-0.87)}{0.09\Delta T_t} - \underset{(-2.12)}{0.14T_{t-2}} + \underset{(3.01)}{0.19T_{t-3}} + \underset{(5.6)}{1.24\Delta CL_t} + \underset{(12.03)}{0.85D_{t-1}}$$

$$- \underset{(-0.3)}{0.03D_{t-2}} - \underset{(-0.36)}{0.02D_{t-3}} + \underset{(0.65)}{0.058246\varepsilon_{t-1}} \quad (SE(\varepsilon_{t-1}) = 0.089865).$$

$R^2 = 0.9769$
$\hat{S}E = 4087777$
$\hat{\rho} = -0.0115$
$DW = 2.0149$

A better estimator for ρ is $\hat{\hat{\rho}} = -0.108435 + 0.058246 = -0.0502$ and

$t = \frac{-0.0502}{0.089865} = -0.5585$; thus $|t| < 1.96$. The first-order autocorrelation in the equation using 2SLS is not significantly different from zero.

Part III. Deposit Supply Function Under Model II Using Lagged Reserve Accounting

Single equation approach

Equation (92) the first-difference of the deposit supply (ΔD) equation from Model II under LRA is estimated in this part. The term RL is added to the other explanatory variables. The estimation procedures used in this part are the same as the procedures used in Part II of this chapter. The resulting estimations for equation (92) are shown as:

(92-1)
$$\Delta D_t = \underset{(0.43)}{50.89} + \underset{(0.65)}{167.13i_t} - \underset{(-1.5)}{572.09d_t} + \underset{(5.75)}{1.70UR_t} - \underset{(-5.69)}{1.69UR_{t-1}}$$
$$- \underset{(-1.29)}{0.14\Delta T_t} - \underset{(-3.11)}{0.41T_{t-2}} + \underset{(3.37)}{0.45T_{t-3}} + \underset{(5.72)}{1.25\Delta CL_t}$$
$$- \underset{(-2.75)}{0.17D_{t-2}} - \underset{(-0.2)}{0.01D_{t-3}} + \underset{(2.59)}{0.40CD_{t-2}} - \underset{(-2.35)}{0.37CD_{t-3}}$$
$$+ \underset{(0.02)}{0.005U_{t-2}} - \underset{(-0.1)}{0.02U_{t-3}} + \underset{(0.28)}{0.004RL_t}$$

$R^2 = 0.4268$

SE = 4178256

$\hat{\rho} = -0.1986$

DW = 2.3911

(92-2)
$$\Delta D_t = \underset{(0.65)}{166.86i_t} - \underset{(-1.49)}{567.28d_t} + \underset{(5.77)}{1.7UR_t} - \underset{(-5.71)}{1.69UR_{t-1}}$$

$$- \underset{(-1.27)}{0.13\Delta T_t} - \underset{(-3.10)}{0.41T_{t-2}} + \underset{(3.37)}{0.45T_{t-3}} + \underset{(5.75)}{1.25\Delta CL_t}$$

$$- \underset{(-2.77)}{0.17D_{t-2}} - \underset{(-0.22)}{0.01D_{t-3}} + \underset{(2.59)}{0.4CD_{t-2}} - \underset{(-2.36)}{0.36CD_{t-3}}$$

$$+ \underset{(0.002)}{0.0004U_{t-2}} - \underset{(-0.08)}{0.02U_{t-3}}$$

$R^2 = 0.4270$

SE = 4155738

$\hat{\rho} = -0.1989$

DW = 2.3917

(92-3) $$\Delta D_t = \underset{(0.64)}{164.22i_t} - \underset{(-1.66)}{602.81d_t} + \underset{(5.83)}{1.71UR_t} - \underset{(-5.72)}{1.68UR_{t-1}} - \underset{(-1.46)}{0.14\Delta T_t}$$

$$- \underset{(-3.3)}{0.42T_{t-2}} + \underset{(3.56)}{0.45T_{t-3}} + \underset{(6.21)}{1.28\Delta CL_t} - \underset{(-2.77)}{0.17D_{t-2}}$$

$$- \underset{(-0.17)}{0.01D_{t-3}} + \underset{(2.66)}{0.4CD_{t-2}} - \underset{(-2.41)}{0.37CD_{t-3}}$$

$R^2 = 0.4268$

SE = 4131545

$\hat{\rho} = -0.1955$

DW = 2.2181

(92-4) $$\Delta D_t = \underset{(0.06)}{8.23} - \underset{(-2)}{519.27i_t} + \underset{(0.83)}{325.37d_t} + \underset{(6.11)}{2.24UR_t} - \underset{(-4.81)}{1.83UR_{t-1}}$$

$$- 0.28\Delta T_t - 11.37(\tau 1T_{t-2}) + 11.33(\tau 1T_{t-3})$$
$$(-2.68) \qquad (-3.31) \qquad (3.36)$$

$$+ 1.46\Delta CL_t - 1.59(\delta D_{t-2}) + 0.5(\delta D_{t-3})$$
$$(6.77) \qquad (-3.5) \qquad (1.09)$$

$$+ 8.48(\tau 1CD_{t-2}) - 9.25(\tau 1CD_{t-3}) + 1.06(\omega U_{t-2})$$
$$(2.13) \qquad (-2.3) \qquad (.56)$$

$$- 0.1(\omega U_{t-3}) + 0.5RL_t$$
$$(-0.14) \qquad (1.09)$$

$R^2 = 0.3852$

SE = 4481587

$\hat{\rho} = -0.1081$

DW = 2.2085

(92-5) $$\Delta D_t = - 517.01i_t + 323.34d_t + 2.24UR_t - 1.83UR_{t-1}$$
$$(-2.02) \qquad (0.83) \qquad (6.16) \qquad (-4.85)$$

$$- 0.27\Delta T_t - 11.36(\tau 1T_{t-2}) + 11.31(\tau 1T_{t-3})$$
$$(-2.68) \qquad (-3.32) \qquad (3.38)$$

$$+ 1.46\Delta CL_t - 1.59(\delta D_{t-2}) + 0.5(\delta D_{t-3})$$
$$(6.78) \qquad (-3.52) \qquad (1.09)$$

$$+ 8.47(\tau 1CD_{t-2}) - 9.25(\tau 1CD_{t-3}) + 1.06(\omega U_{t-2})$$
$$(2.13) \qquad (-2.3) \qquad (0.56)$$

$$- 0.11(\omega U_{t-3}) + 0.5RL_t$$
$$(-0.15) \qquad (1.09)$$

$R^2 = 0.3859$

$SE = 4467685$

$\hat{\rho} = -0.1082$

$DW = 2.2085$

(92-6) $$\Delta D_t = -\ 17.03 - 333.53i_t + 40.37d_t + 2.17UR_t - 181UR_{t-1}$$
(-0.13) (-1.44) (.13) (5.95) (-4.82)

$$-\ 0.31\Delta T_t - 2.31(\tau 1T_{t-2} + \tau 1CD_{t-2})$$
(-3.04) (-2.41)

$$+\ 2.03(\tau 1T_{t-3} + \tau 1CD_{t-3}) + 1.52\Delta CL_t$$
(2.15) (7.06)

$$-\ 1.48(\delta D_{t-2}) + 0.5(\delta D_{t-3}) + 1.23(\omega U_{t-2})$$
(-3.24) (1.1) (0.65)

$$-\ 0.15(\omega U_{t-3}) + 0.51RL_t$$
(-0.2) (1.11)

$R^2 = .3689$

$SE = 4571850$

$\hat{\rho} = -0.1141$

$DW = 2.2222$

(92-7) $$\Delta D_t = -336.81i_t + 41.96d_t + 2.17UR_t - 1.81UR_{t-1}$$
(-1.46) (0.13) (5.97) (-4.88)

$$-\ 0.31\Delta T_t - 2.32(\tau 1T_{t-2} + \tau 1CD_{t-2})$$
(-3.07) (-2.42)

$$+\ 2.05(\tau 1T_{t-3} + \tau 1CD_{t-3}) + 1.52\Delta CL_t - 1.47(\delta D_{t-2})$$
(2.2) (7.07) (-3.25)

$$+ 0.51(\delta D_{t-3}) + 1.22(\omega U_{t-2}) - 0.14(\omega U_{t-3}) + 0.51RL_t$$
$$(1.12) \qquad (0.65) \qquad (-0.19) \qquad (1.13)$$

$R^2 = 0.3696$

$SE = 4557950$

$\hat{\rho} = -0.1139$

$DW = 2.2219$

Equations (92-1) and (92-2) show two versions of the estimations for equation (92) from Model II under LRA in Part III of Chapter III. The version with no intercept and RL term gives a better performance. The signs of the estimated coefficients on the explanatory variables are as expected, except for variables D_{t-3}, CD_{t-2}, CD_{t-3}, U_{t-2}, and U_{t-3}. The estimated coefficients on the variables D_{t-3}, U_{t-2}, U_{t-3}, ΔT_t, i_t and d_t are not significantly different from zero. Equation (92-3) shows the estimation of the resulting equation from Model II under LRA with the inclusion of only CD to the Model I. Equation (92-3) indicates a better performance in explaining ΔD_t than equation (92-2). Except for the estimated coefficients on CD_{t-2} and CD_{t-3} and D_{t-3}, the other coefficients have the expected signs. Equations (92-4) and (92-5) show the estimations for equation (92) taking the variations of the deseasonalized required reserve ratios into account. As mentioned before, $\tau 1$ is the composite ratio for the required reserves against the time deposit and CD. To correct the measurement errors in variables $\tau 1T_{t-2}$, $\tau 1T_{t-3}$, $\tau 1CD_{t-2}$, and $\tau 1CD_{t-3}$, the required reserve against total time deposit including CD, namely ($\tau 1T_{t-2}+\tau 1CD_{t-2}$) and $\tau 1(T_{t-3}+CD_{t-3})$, are used as the explanatory variables in equations (92-6) and (92-7).

Equations (92-2) and (92-3) which have smaller SE and larger R^2 than the other estimations are chosen for further modifications. In both equations, the signs of the estimated coefficients for CD contradict the results of the theory in Model II under LRA. The way that CD is integrated into Model II under LRA channels the effect of the change in the outstanding level of CD upon the deposit supply through the change in the banking reserves. But the conflicting results on the signs of the coefficients for CD_{t-2} or CD_{t-3} might suggest that additional direct effects by the lagged CD's on the first difference of the deposit supply exist. Model II under LRA has not considered these effects.

Equations (92-7) and (92-8) show the corrected form of equations (92-2) and (92-3) for the first-order autocorrelation, respectively.

(92-7)
$$\Delta D_t = \underset{(1.11)}{243.48 i_t} - \underset{(-1.83)}{586.55 d_t} + \underset{(5.9)}{1.72 UR_t} - \underset{(-5.9)}{1.72 UR_{t-1}}$$
$$- \underset{(-0.35)}{0.04 \Delta T_t} - \underset{(-2.81)}{0.34 T_{t-2}} + \underset{(3.15)}{0.38 T_{t-3}} + \underset{(5.5)}{1.13 \Delta CL_t}$$
$$- \underset{(-3.82)}{0.23 D_{t-2}} + \underset{(0.44)}{0.03 D_{t-3}} + \underset{(2.53)}{0.35 CD_{t-2}} - \underset{(-2.29)}{0.32 CD_{t-3}}$$
$$+ \underset{(0.003)}{0.0008 U_{t-2}} - \underset{(-0.21)}{0.54 U_{t-3}} - \underset{(-0.56)}{0.032793 \varepsilon_{t-1}}$$

$R^2 = .4509$

SE = 3977061

$\hat{\rho} = .0022$

DW = 1.9883

From equation (92-2) $\hat{\hat{\rho}} = -0.1996-0.032793 = -0.2324$, and

$$t = \frac{-0.2324}{0.058702} = -3.9582$$

where $SE(\varepsilon_{t-1}) = 0.058702$ in equation (92-7). As seen above, $|t| > 1.96$ which shows a significant first-order autocorrelation at a 5% level of significance for equation (92-2). The first-order autocorrelation in equation (92-7) is not significantly different from zero. The second-order autocorrelation in equation (92-2) is not expected to be significantly different from zero. Therefore, equation (92-7) is used instead of (92-2) with a better performance in terms of smaller SE and larger R^2.

$$(92\text{-}8) \quad \underset{}{\Delta D_t} = \underset{(1.06)}{233.72 i_t} - \underset{(-2.21)}{680.72 d_t} + \underset{(6.04)}{1.75 UR_t} - \underset{(-5.91)}{1.71 UR_{t-1}} - \underset{(-0.8)}{0.076 \Delta T_t}$$

$$- \underset{(-3.11)}{0.36 T_{t-2}} + \underset{(3.42)}{0.4 T_{t-3}} + \underset{(6.3)}{1.21 \Delta CL_t} - \underset{(-3.78)}{0.23 D_{t-2}}$$

$$+ \underset{(0.6)}{0.04 D_{t-3}} + \underset{(2.67)}{0.36 CD_{t-2}} - \underset{(-2.38)}{0.33 CD_{t-3}} - \underset{(-0.47)}{0.027162 \varepsilon_{t-1}}$$

$R^2 = 0.4484$

$SE = 3967457$

$\hat{\rho} = 0.0051$

$DW = 1.9826$

From equation (92-3) $\hat{\hat{\rho}} = -0.1962-0.027162 = -0.2234$, and

$$t = \frac{-0.2234}{0.057747} = -3.868$$

where $SE(\varepsilon_{t-1}) = 0.057747$ in equation (92-8). Thus, $|t| > 1.96$, which shows a significant first-order autocorrelation at a 5% level of significance for equation (92-3). The second-order autocorrelation for equation (92-3) is not expected to be significantly different from zero. As seen above, the first-order autocorrelation for equation (92-8) is not significantly different from zero. Therefore, equation (92-8) is chosen instead of equation (92-3) which has a better performance in terms of smaller SE and larger R^2.

In both equations (92-7) and (98-8), signs of the estimated coefficients are as expected except for the coefficients for variables CD_{t-2}, CD_{t-3}, U_{t-2}, and U_{t-3}.

Simultaneous equation approach

As in Part II of this chapter, equations (92-2) and (92-3) are reestimated by use of 2SLS procedure, taking into account the same functional form for the demand for deposit as utilized in Part II of this chapter. The system of equations used in 2SLS procedure is the same as the system presented in Part II except that the deposit supply equation is replaced by equation (92-2) and then (92-3).

For equation (92-2) the result is shown below in equation (92-9).

$$\begin{aligned}(92\text{-}9)\qquad \Delta D_t = &- \underset{(-1.19)}{707.31 i_t} + \underset{(0.69)}{532.82 d_t} + \underset{(5.08)}{1.57 UR_t} - \underset{(-5.83)}{1.8 UR_{t-1}} - \underset{(-1.77)}{0.2 \Delta T_t} \\ &- \underset{(3.44)}{0.5 T_{t-2}} + \underset{(3.69)}{0.54 T_{t-3}} + \underset{(5.85)}{1.4 \Delta CL_t} - \underset{(-2.55)}{0.16 D_{t-2}} + \underset{(0.44)}{0.03 D_{t-3}} \\ &+ \underset{(2.98)}{0.51 CD_{t-2}} - \underset{(-2.79)}{0.49 CD_{t-3}} - \underset{(-0.12)}{0.03 U_{t-2}} + \underset{(0.06)}{0.01 U_{t-3}}\end{aligned}$$

$R^2 = 0.4194$

$SE = 4306457$

$\hat{\rho} = -0.1474$

$DW = 2.2844$

Using the simultaneous equation approach, equation (92-3) is reestimated in equation (92-10) below.

$$(92\text{-}10) \quad \Delta D_t = \underset{(-1.48)}{-\ 908.24 i_t} + \underset{(0.95)}{753.84 d_t} + \underset{(4.97)}{1.55 UR_t} - \underset{(-5.87)}{1.82 UR_{t-1}}$$

$$- \underset{(-2.06)}{0.23 \Delta T_t} - \underset{(-3.71)}{0.52 T_{t-2}} + \underset{(3.95)}{0.56 T_{t-3}} + \underset{(6.33)}{1.45 \Delta CL_t}$$

$$- \underset{(-2.5)}{0.16 D_{t-2}} + \underset{(0.62)}{0.04 D_{t-3}} + \underset{(3.14)}{0.53 CD_{t-2}} - \underset{(-2.95)}{0.51 CD_{t-3}}$$

$R^2 = 0.4157$

$SE = 4357480$

$\hat{\rho} = -0.1261$

$DW = 2.2406$

With the same procedure utilized in Part II, equation (92-9) is corrected for the first-order autocorrelation in the context of the simultaneous equation approach. The result is shown below in equation (92-11).

$$(92\text{-}11) \quad \Delta D_t = \underset{(0.66)}{368.57 i_t} - \underset{(-1.05)}{748 d_t} + \underset{(5.77)}{1.73 UR_t} - \underset{(-5.64)}{1.7 UR_{t-1}} - \underset{(-0.25)}{0.03 \Delta T_t}$$

$$- 0.34T_{t-2} + 0.38T_{t-3} + 1.12\Delta CL_t - 0.23D_{t-2}$$
$$(-2.47) \quad (2.76) \quad (4.78) \quad (-3.66)$$

$$+ 0.014D_{t-3} + 0.34CD_{t-2} - 0.3CD_{t-3} - 0.02U_{t-2}$$
$$(0.22) \quad (2.15) \quad (-1.89) \quad (-0.06)$$

$$- 0.04U_{t-3} - 0.067264\varepsilon_{t-1}$$
$$(-0.14) \quad (-1.06)$$

$R^2 = 0.4454$

SE = 3988983

$\hat{\rho} = -0.0056$

DW = 2.0044

From equation (92-9) $\hat{\hat{\rho}} = -0.157186-0.067264 = -0.2245$, and

$$t = \frac{-0.2245}{0.063457} = -3.537,$$

where $SE(\varepsilon_{t-1}) = 0.063457$ in equation (92-11). The first-order autocorrelation is significantly different from zero at a 5% level of significance. The second-order autocorrelation is not expected to be significantly different from zero. Therefore, equation (92-11) is used instead of equation (92-9) because of a smaller SE and a larger R^2.

Similarly, the corrected form for autocorrelation of equation (92-10) is shown below in equation (92-12).

(92-12) $$\Delta D_t = 440.81i_t - 942.13d_t + 1.75UR_t - 1.66UR_{t-1} - 0.06\Delta T_t$$
$$(0.75) \quad (-1.26) \quad (5.88) \quad (-5.5) \quad (-0.54)$$

$$- 0.35T_{t-2} + 0.39T_{t-3} + 1.18\Delta CL_t - 0.22D_{t-2}$$
$$(-2.65) \quad (2.9) \quad (5.31) \quad (-3.57)$$

$$+ 0.01D_{t-3} + 0.34CD_{t-2} - 0.3CD_{t-3} - 0.087618\varepsilon_{t-1}$$
$$(0.24) \quad (2.17) \quad (-1.87) \quad (-1.35)$$

R^2 = 0.4417

SE = 3985969

$\hat{\rho}$ = -0.0015

DW = 1.9965

From equation (92-10) $\hat{\hat{\rho}}$ = -0.129366-0.087618 = -0.217, and

$$t = \frac{-0.217}{0.06506} = -3.3351$$

where

SE(ε_{t-1}) = 0.06506 in equation (92-12).

The first order autocorrelation is significantly different from zero at a 5% level of significance. The second-order autocorrelation is not expected to be significantly different from zero. Therefore, equation (92-12) is used instead of equation (92-10) because of a smaller SE and a larger R^2.

The endogenous variables in the system of simultaneous equations are the same in all cases.

The exogenous variables in the system including equation (92-2) are shown below.

d_t, UR_t, UR_{t-1}, ΔT_t, T_{t-2}, T_{t-3}, D_{t-2}, D_{t-3}, $r_{S\&T_t} Y_t$, ΔCL_t,

Y_t, $N_t P_t$, TD^d_{t-1}, IBD^m_t, $Float_t$, FD_t, DG_t, S_t, D_{t-1}, CD_{t-2},

CD_{t-3}, U_{t-2}, U_{t-3}, ε_{t-1}.

The exogenous variables in the system including equation (92-3) are shown below.

$$d_t,\ UR_t,\ UR_{t-1},\ \Delta T_t,\ T_{t-2},\ T_{t-3},\ D_{t-2},\ D_{t-3},\ r_{S\&T_t} Y_t,\ \Delta CL_t,$$

$$Y_t,\ N_t P_t,\ TD^d_{t-1},\ IBD^m_t,\ Float_t,\ FD_t,\ DG_t,\ S_t,\ D_{t-1},$$

$$CD_{t-2},\ CD_{t-3},\ \varepsilon_{t-1}.$$ [1]

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[1]The functional form expressed in equation (92) of Model II under LRA is used to estimate the level of the deposit supply using no unity constraints on the coefficient of the one lagged deposit supply as an explanatory variable. Using OLS, the results are shown below.

$$\begin{aligned} D_t = {} & \underset{(1.19)}{299.19 i_t} - \underset{(-1.71)}{634.1 d_t} + \underset{(5.61)}{1.62 UR_t} - \underset{(-5.47)}{1.59 UR_{t-1}} - \underset{(-0.11)}{0.01 \Delta T_t} \\ & - \underset{(-3.4)}{0.44 T_{t-2}} + \underset{(3.73)}{0.48 T_{t-3}} + \underset{(50.8)}{1.1 \Delta CL_t} - \underset{(-0.03)}{0.002 D_{t-2}} - \underset{(-0.62)}{0.04 D_{t-3}} \\ & + \underset{(17.94)}{0.81 D_{t-1}} + \underset{(2.7)}{0.41 CD_{t-2}} - \underset{(-2.49)}{0.38 CD_{t-3}} - \underset{(-0.75)}{0.19 U_{t-2}} + \underset{(0.5)}{0.12 U_{t-3}} \end{aligned}$$

$R^2 = 0.9730$

$SE = 3955316$

$\hat{\rho} = -0.0406$

$DW = 2.0750$

Part IV. Deposit Supply Function under Model III Using Lagged Reserve Accounting

Single equation approach

Equation (98), the first-difference of the deposit supply (ΔD) equation from Model III under LRA, is estimated in this part. The estimation procedure used in this part is the same as the procedures used in Parts II and III.

For equation (98) the resulting estimations are shown as:

(98-1)
$$D_t = \underset{(-0.04)}{-5.55} + \underset{(0.88)}{254.43 i_t} - \underset{(-0.01)}{7.22 d_t} - \underset{(-1.08)}{212.63 r_{F_t}}$$
$$- \underset{(-1.88)}{0.21 \Delta T_t} + \underset{(6.92)}{1.54 \Delta CL_t} - \underset{(-1.46)}{0.21 UR_{t-1}} + \underset{(3.18)}{0.04 T_{t-3}}$$
$$- \underset{(-5.05)}{0.19 D_{t-3}} + \underset{(1.29)}{0.03 CD_{t-3}} - \underset{(-1.66)}{0.11 U_{t-3}} + \underset{(0.37)}{0.01 RL_t}$$

$R^2 = 0.35$

$SE = 4706362$

$\hat{\rho} = -0.2012$

$DW = 2.4007$

(98-2)
$$\Delta D_t = \underset{(0.89)}{256.56 i_t} - \underset{(-0.03)}{17.65 d_t} - \underset{(-1.08)}{120.15 r_{F_t}} - \underset{(-1.94)}{0.21 \Delta T_t}$$
$$+ \underset{(6.99)}{1.55 \Delta CL_t} - \underset{(-1.46)}{0.21 UR_{t-1}} + \underset{(3.26)}{0.04 T_{t-3}} - \underset{(-5.10)}{0.19 D_{t-3}}$$

$$+ \underset{(1.32)}{0.03CD_{t-3}} - \underset{(-1.65)}{0.11U_{t-3}}$$

$R^2 = 0.3468$

$SE = 4679540$

$\hat{\rho} = -0.2000$

$DW = 2.3982$

(98-3) $$\Delta D_t = \underset{(1.14)}{153.25} - \underset{(-1.42)}{417.59i_t} + \underset{(3.17)}{1492.55d_t} - \underset{(-2.81)}{630.21r_{F_t}}$$

$$- \underset{(-3.42)}{0.37\Delta T_t} + \underset{(8.01)}{1.74\Delta CL_t} - \underset{(-2.62)}{0.93UR_{t-1}} + \underset{(2.71)}{1.89(\tau 1T_{t-3})}$$

$$+ \underset{(0.71)}{0.29(\delta D_{t-3})} - \underset{(-0.85)}{0.54(\tau 1CD_{t-3})} + \underset{(0.35)}{0.56(\omega U_{t-3})}$$

$$+ \underset{(0.73)}{0.29RL_t}$$

$R^2 = 0.3102$

$SE = 4966236$

$\hat{\rho} = -0.1133$

$DW = 2.2231$

(98-4) $$\Delta D_t = - \underset{(-1.46)}{427.67i_t} + \underset{(3.23)}{1328.75d_t} - \underset{(-3.17)}{525.46r_{F_t}} - \underset{(-3.33)}{0.36\Delta T_t} + \underset{(8)}{1.74\Delta CL_t}$$

$$- \underset{(-4.81)}{0.67UR_{t-1}} + \underset{(3)}{1.53(\tau 1T_{t-3})} - \underset{(-0.5)}{0.01(\delta D_{t-3})} - \underset{(-0.97)}{0.61(\tau 1CD_{t-3})}$$

$$- 0.05(\omega U_{t-3})$$
$$(-0.08)$$

R^2 = .3080

SE = 4957532

$\hat{\rho}$ = -0.1152

DW = 2.2273

Equations (98-1) and (98-2) show the estimations of equation (98) from Model III under LRA in Chapter III, Part IV, using OLS. All variables are regressed on intercept and seasonal dummies. Hence, the intercept term has a very small explanatory power in equation (98-1). The deseasonalized required reserve ratios have almost no variation during the span of the time period of this study. Thus, the estimated coefficient for RL_t is also insignificantly different from zero in equation (98-1). Therefore, equation (98-2) is formulated by elimination of the intercept and RL_t terms from equation (98-1). Equations (98-3) and (98-4) show the estimations of equation (98) taking into account the variations of the required reserve ratios. In Part II, more explanation is presented about these versions of the estimations. Equation (98-2) with the lowest SE is chosen for further analysis. As noticed in the estimation (98-2) compared to equation (98) in Part IV of Chapter III, variable r_{RP_t}, the RP market interest rate, is eliminated. There are no available time series data on this variable. On the other hand, r_{RP_t} keeps track of FFR or r_{F_t} very closely. Therefore, r_{F_t} alone can explain the variations of r_{RP_t} by its variation. Hence, this problem is not of significance.

Except for the signs of the estimated coefficients for D_{t-3} and U_{t-3}, the other variables have the expected signs. Under Model III using LRA, the policy variable is r_{F_t}, however, as seen in estimation (98-2) its coefficient is not significantly different from zero.

Equation (98-2) corrected for the first-order autocorrelation using GLS approach, is shown below in equation (98-5).

$$\text{(98-5)} \quad \Delta D_t = \underset{(1.37)}{334.79 i_t} - \underset{(-0.49)}{209.62 d_t} - \underset{(-0.86)}{140.88 r_{F_t}} - \underset{(-0.58)}{0.06 \Delta T_t} + \underset{(6.16)}{1.32 \Delta CL_t}$$

$$- \underset{(-1.30)}{0.16 UR_{t-1}} + \underset{(4.05)}{0.04 T_{t-3}} - \underset{(-6.45)}{0.2 D_{t-3}} + \underset{(1.54)}{0.029 CD_{t-3}}$$

$$- \underset{(-2.17)}{0.12 U_{t-3}} - \underset{(-0.43)}{0.024521 \varepsilon_{t-1}}$$

$$(SE(\varepsilon_{t-1}) = 0.056727)$$

$R^2 = 0.3728$

$SE = 4486496$

$\hat{\rho} = -0.0052$

$DW = 2.0071$

From equation (98-2) $\hat{\hat{\rho}} = -0.199959 - .024621 = -0.2246$, and

$$t = \frac{-0.2246}{0.056727} = -3.959.$$

As seen above, $|t| > 1.96$; hence, a significant first-order autocorrelation at a 5% level of significance exists in equation (98-2). The second-order autocorrelation in equation (98-2) is expected to be not significantly different from zero. Equation (98-5) in the corrected form, with smaller SE and larger R^2 is utilized for further analysis.

Simultaneous equation approach

Equation (98-2) is reestimated using the system of simultaneous equations. The system of the equations are the same as the ones utilized in Part II, except that the deposit supply equation is replaced by equation (98-2). 2SLS is used as an estimation procedure. The endogenous variables seen are the same as those in Model I under LRA in Part II of this chapter. The exogenous variables are as follows:

$$d_t,\ r_{F_t},\ UR_{t-1},\ \Delta T_t,\ T_{t-3},\ D_{t-3},\Delta CL_t,\ r_{S\&T_t}Y_t,\ N_tP_t,\ Y_t,$$

$$TD^d_{t-1},\ IBD^m_t,\ Float_t,\ FD_t,\ DG_t,\ S_t,\ D_{t-1},\ CD_{t-3},\ U_{t-3},\ \varepsilon_{t-1}.$$

The term ε_{t-1} is included in the system when the system of equations is corrected for autocorrelation. ε_{t-1} is the lagged value of the residual of the deposit supply equation when 2SLS is used in the above system.

Taking into account the above system, equation (98-2) is reestimated in equation (98-6).

$$(98\text{-}6)\qquad \Delta D_t = -\underset{(-1.46)}{1199.04 i_t} + \underset{(1.29)}{929.69 d_t} + \underset{(0.64)}{184.78 r_{F_t}} - \underset{(-2.35)}{0.28\Delta T_t} + \underset{(6.98)}{1.7\Delta CL_t}$$

$$-\underset{(-2.33)}{0.47 UR_{t-1}} + \underset{(3.47)}{0.049 T_{t-3}} - \underset{(-2.71)}{0.13 D_{t-3}} + \underset{(0.94)}{0.023 CD_{t-3}}$$

$$-\underset{(-0.77)}{0.056 U_{t-3}}$$

$R^2 = 0.3316$

SE = 5049597

$\hat{\rho} = -0.1063$

DW = 2.2068

Using the procedure explained in Part I, Serial Correlation Section of this chapter, equation (98-6) is corrected for the first-order autocorrelation in the context of simultaneous equation approach. The result is shown in equation (98-7).

$$\text{(98-7)} \quad \Delta D_t = -\underset{(-0.38)}{264 i_t} + \underset{(0.35)}{225.54 d_t} - \underset{(-0.02)}{5.54 r_{F_t}} - \underset{(-1.038)}{0.13 \Delta T_t}$$

$$+ \underset{(5.96)}{1.45 \Delta CL_t} - \underset{(-1.51)}{0.27 UR_{t-1}} + \underset{(3.94)}{0.05 T_{t-3}} - \underset{(-4.13)}{0.18 D_{t-3}}$$

$$+ \underset{(1.3)}{0.03 CD_{t-3}} - \underset{(-1.6)}{0.1 U_{t-3}} - \underset{(-0.63)}{0.04 \varepsilon_{t-1}}$$

$$(SE(\varepsilon_{t-1}) = 0.067731)$$

$R^2 = .3609$

SE = 4579039

$\hat{\rho} = -0.0123$

DW = 2.0200

From equation (98-6), $\hat{\hat{\rho}} = -0.144692-0.042439 = -0.1871$ and

$$t = \frac{-0.1871}{0.067731} = -2.7629.$$

$|t| > 1.96$ suggests that the first-order autocorrelation exists in equation (98-6) and significantly differs from zero at a 5% level of significance. The second-order autocorrelation is not expected to be signigicantly different from zero in equation (98-6). Therefore, equation (98-7) is utilized for further analysis instead of equation (98-6). The SE of equation (98-7) is smaller and its R^2 is larger than equation (98-6).

In equation (98-7), the signs of the coefficients for i_t, D_{t-3}, and U_{t-3} contradict the results from Model III under LRA, but the other signs are as expected. The problem in equation (98-7) is that most of the variables are not significantly different from zero. This problem is also seen in equation (98-5). It means that in Model III under LRA, some of the important explanatory variables which explain the variation of ΔD_t have been eliminated. Appearance of the three highly correlated explanatory variables, namely i_t, d_t, r_{F_t}, might be another explanation for the poor performance of Model III under LRA. Simple correlation coefficients for the above three variables are as follows:

Simple correlation coefficient of $(i_t, d_t) = 0.89557$;

Simple correlation coefficient of $(i_t, r_{F_t}) = 0.9316$;

Simple correlation coefficient of $(d_t, r_{F_t}) = 0.8939$.[1]

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[1]The functional form expressed in equation (98) of Model III under LRA is used to estimate the level of the deposit supply using no unity constraints on the coefficient of the one lagged deposit supply as an explanatory variable. The results are shown below.

1) Using OLS:

$$\underset{}{D_t} = \underset{(0.51)}{63.61} + \underset{(1.35)}{349.15 i_t} + \underset{(3.77)}{1112.43 d_t} - \underset{(-4.58)}{1049.09 r_{F_t}} - \underset{(-4.11)}{0.44 \Delta T_t} + \underset{(9.06)}{1.93 \Delta CL_t}$$

$$- \underset{(-2.52)}{0.79 FR_{t-1}} + \underset{(82.64)}{0.97 D_{t-1}}$$

$R^2 = 0.9643$ $\quad\hat{\rho} = -0.0641$

$SE = 5071753$ $\quad DW = 2.1254$

(Footnote continued on following page).

(Footnote continued from preceding page).

2) Using GLS for testing and correcting for the first-order autocorrelation

$$D_t = \underset{(0.24)}{35.95} + \underset{(1.24)}{368.33i_t} + \underset{(2.36)}{855.77d_t} - \underset{(-3.59)}{940.9r_{F_t}} - \underset{(-3.9)}{0.43\Delta T_t}$$

$$+ \underset{(8.96)}{1.94\Delta CL_t} - \underset{(-1.9)}{0.67FR_{t-1}} + \underset{(59.14)}{0.99D_{t-1}} - \underset{(-3.61)}{0.234002\varepsilon_{t-1}}$$

$$(SE(\varepsilon_{t-1}) = 0.06486)$$

$R^2 = 0.9515$ $\hat{\rho} = 0.0058$
SE = 5086442 DW = 1.9878

A better estimator for ρ is $\hat{\hat{\rho}} = +0.150409-0.234002 = -0.0836$ and $t = \frac{-0.0836}{0.06486} = -1.2888$; thus $|t| < 1.96$. The first-order autocorrelation in equation using OLS is not significantly different from zero.

3) Using 2SLS utilizing the demand equation for the demand deposit used in this part:

$$D_t = \underset{(0.35)}{50.65} + \underset{(4.14)}{2912.81i_t} + \underset{(0.91)}{350.22d_t} - \underset{(-5.62)}{2417.15r_{F_t}} - \underset{(-3.1)}{0.38\Delta T_t}$$

$$+ \underset{(7.2)}{1.77\Delta CL_t} - \underset{(-3.83)}{1.57FR_{t-1}} + \underset{(72.58)}{0.97D_{t-1}}$$

$R^2 = 0.9542$ $\hat{\rho} = 0.1426$
SE = 6592422 DW = 1.7060

4) Using the procedure explained in Serial Correlation Section of Part I of this chapter, the above equation is tested and corrected for the first-order autocorrelation in context with the simultaneous equation approach.

$$D_t = \underset{(0.39)}{53.88} + \underset{(4.34)}{2923.97i_t} + \underset{(0.58)}{219.48d_t} - \underset{(-5.65)}{2330.32r_{F_t}} - \underset{(-2.59)}{0.33\Delta T_t}$$

$$+ \underset{(6.61)}{1.66\Delta CL_t} - \underset{(-3.69)}{1.47FR_{t-1}} + \underset{(75.99)}{0.97D_{t-1}} - \underset{(-0.19)}{0.012081\varepsilon_{t-1}}$$

$$(SE(\varepsilon_{t-1}) = 0.064795)$$

A better estimator for ρ is $\hat{\hat{\rho}} = -0.061863-0.012081 = -0.0739$ and $t = \frac{-0.0739}{0.064795} = -1.1412$; thus $|t| < 1.96$. The first-order autocorrelation in the equation using 2SLS, is not significantly different from zero.

Part V. Simulation

To test the ability of the models to generate the historical values of the key variables, namely i and D, a Gauss-Seidel simulation technique is utilized.

The system of simultaneous equations under Model I including equation (44-7) and corrected for autocorrelation is selected for the purpose of simulation. This system is shown below:

1) $\Delta D_t^* = 351.01 i_t^* - 546.5 d_t^* + 1.76 UR_t^* - 1.61 UR_{t-1}^*$

$$- 0.13 \Delta T_t^* - 0.12 T_{t-2}^* + 0.17 T_{t-3}^* + 1.27 \Delta CL_t^*$$

$$- 0.21 D_{t-2}^* + 0.02 D_{t-3}^* - 0.07536 \varepsilon_{t-1}.$$

2) $TD_t^d = - 0.00029(i_t Y_t) + 0.0033(r_{S\&T_t} Y_t) + 0.03 Y_t - 0.0016(N_t P_t)$

$$+ 0.87 TD_{t-1}^d.$$

3) $D1_t^m \equiv D_t - IBD_t^m - Float_t + FD_t.$

4) $TD_t^d \equiv S_t D1_t^m - DG_t.$

5) $i_t Y_t \equiv i_t X Y_t.$

6) $S_t D1_t^m \equiv S_t \times D1_t^m.$

7) $\Delta D_t \equiv D_t - D_{t-1}.$

8) $\Delta D_t^* \equiv \Delta D_t + 0.122 \Delta D_{t-1}.$

9) $i_t^* \equiv i_t + 0.122 i_{t-1}$

10) $D^*_{t-2} \equiv D_{t-2} + 0.122D_{t-3}$.

11) $D^*_{t-3} \equiv D_{t-3} + 0.122D_{t-4}$.

The variables with superscripts (*) are the transformed variables for autocorrelation correction, where $\hat{\rho} = -0.122$, is the value used for transforming the values of the variables.

The above system can be solved and reduced to a system of six equations and identities:

1) $D_t = 351.01i_t + 42.82i_{t-1} - 546.5d^*_t + 1.76UR^*_t - 1.16UR^*_{t-1} - 0.13\Delta T^*_t$
$- 0.12T^*_{t-2} + 0.17T^*_{t-3} + 1.27\Delta CL^*_t - 0.07\varepsilon_{t-1} + 0.878D_{t-1}$
$- 0.088D_{t-2} - 0.046D_{t-3} - 0.0024D_{t-4}$.

2) $TD^d_t = - 0.00029(i_tY_t) + 0.0033(r_{S\&T_t}Y_t) + 0.03Y_t - 0.0016(N_tP_t)$
$+ 0.87TD^d_{t-1}$.

3) $D1^m_t \equiv D_t - IBD^m_t - Float_t + FD_t$.

4) $TD^d_t \equiv S_tD1^m_t - DG_t$.

5) $S_tD1^m_t \equiv S_t \times D1^m_t$.

6) $i_tY_t \equiv i_t \times Y_t$

The endogenous variables in the above system are D_t, TD^d_t, i_t, (i_tY_t), $D1^m_t$, $(S_tD1^m_t)$. The exogenous variables are d^*_t, UR^*_t, ΔT^*_t, ΔCL^*_t, ε_{t-1},

T^*_{t-2}, $(r_{S\&T_t} Y_t)$, Y_t, $(N_t P_t)$, IBD^m_t, $Float_t$, FD_t, DG_t, S_t

i_{t-1}, D_{t-1}, D_{t-2}, D_{t-3}, D_{t-4}, TD_{t-1} (lagged endogenous variables)

UR^*_{t-1}, T^*_{t-3} (lagged exogenous variables)

The above system of simultaneous equations is nonlinear in some of its variables. The use of the reduced form approach for the simulation purpose is not feasible. For a linear system, the matrix for the reduced form coefficients consists of constant elements.

$$W = B^{-1}X$$

where

W = A matrix for endogenous variable;

X = A matrix for predetermined variables

B^{-1}= A matrix for constant reduced form coefficients.

Some of the elements of the matrix analogous to B^{-1} in the above system are functions of the elements in W. Therefore, it is not feasible to find an explicit functional form for the endogenous variables. Because of the above problem, the system is simulated using the Gauss-Seidel algorithm. By using this technique, the starting values for the endogenous variables are iterated for the solution of the system of equations for the first week. The estimated values of the variables from the first week are taken as the starting values and iterated for the solution of the system of equations for the second week, and so on. A period of one year from July, 1975, is undertaken for simulation. The entire period of 339 weekly observations is not utilized because of the limited capacity of the available Gauss-Seidel program. The values of some variables are too large, hence, all variables except S_t are scaled by $\frac{1}{100}$ and the variable $(i_t Y_t)$ are scaled by $\frac{1}{10000}$. The coefficient for the variable $(i_t Y_t)$ in equation (2) of the above system is multiplied by 100.

To measure the accuracy of the forecast , the Theil's Inequality Coefficient U (or U statistic (RFORM)) is utilized.

$$U = \frac{\sqrt{\Sigma_t (\omega - \hat{\omega})^2}}{\sqrt{\Sigma_t \omega^2} + \sqrt{\Sigma_t \hat{\omega}^2}}$$

where

ω = the acutal value for the endogenous variable

$\hat{\omega}$ = the forecasted value for the endogenous variable.

U statistic approximately measures the average percent error of forecast. It is bounded by zero and one and its value is equal to zero if forecast is perfect.

The values of U statistics for the two key variables D and i are shown below.

$$U_D = 0.299 \quad , \quad U_i = 0.954$$

The results show a weak ability of forecasted values to follow the actual values. This problem is more apparent for the three month Treasury bill interest rate (i).

For the estimation of Model I, the entire period of 339 weekly observations is utilized. As mentioned above, however, a period of the last 52 weeks of the period under this study is employed for simulation. Therefore, the U statistic results, presented above, cannot be taken as an adequate measure for the forecasting ability of Model I over the entire period of this study. Moreover, the system in Model I includes a large number of lagged endogenous variables. In Gauss-Seidel simulation approach, the endogenous variables are estimated in the first period and then fed into the second period as lagged endogenous variables, and so on. Therefore, if there is any error in the first period forecast, it will be magnified in the later periods. This problem, in part, may cause a large forecasting error. The forecasted values of the endogenous variable deviate more from their actual values in later periods. Finally, the weak explanatory power of i in Model I may have been responsible for the high U statistic value for variable i.

The system of simultaneous equations under Model II including equation (92-12) and corrected for autocorrelation is shown below:

1) $$\Delta D_t^* = 440.81 i_t^* - 942.13 d_t^* + 1.75 UR_t^* - 1.66 UR_{t-1}^* - 0.06 \Delta T_t^* - 0.35 T_{t-2}^* + 0.39 T_{t-3}^* + 1.18 \Delta CL_t^* - 0.22 D_{t-2}^* + 0.015 D_{t-3}^* + 0.34 CD_{t-2}^* - 0.3 CD_{t-3}^* - 0.09 \varepsilon_{t-1}.$$

2) $$TD_t^d = - 0.00026(i_t Y_t) + 0.0032(r_{S\&T_t} Y_t) + 0.028 Y_t - 0.0016(N_t P_t) + 0.87 TD_{t-1}^d$$

3) $$D1_t^m \equiv D_t - IBD_t^m - Float_t + FD_t$$

4) $$TD_t^d \equiv S_t D1_t^m - DG_t$$

5) $i_tY_t \equiv i_t \times Y_t$

6) $S_tD1_t^m \equiv S_t \times D1_t^m$

7) $\Delta D_t \equiv D_t - D_{t-1}$

8) $\Delta D_t^* \equiv \Delta D_t + 0.129\Delta D_{t-1}$

9) $i_t^* \equiv i_t + 0.129i_{t-1}$

10) $D_{t-2}^* \equiv D_{t-2} + 0.129D_{t-3}$

11) $D_{t-3}^* \equiv D_{t-3} + 0.129D_{t-4}$

The variables with superscripts(*) are the transformed variables for the autocorrelation correction, where $\hat{\rho} = -0.129$ is the value used for transforming the values of the variables. The above system is reduced to a system of six equations:

1) $$D_t = 440.81i_t + 56.86i_{t-1} - 942.13d_t^* + 1.75UR_t^* - 1.66UR_{t-1}^* - 0.06\Delta T_t^*$$
$$- 0.35T_{t-2}^* + 0.39T_{t-3}^* + 1.18\Delta CL_t^* + 0.34CD_{t-2}^* + 0.3CD_{t-3}^* - 0.09\varepsilon_{t-1}$$
$$+ 0.871D_{t-1} - 0.091D_{t-2} - 0.013D_{t-3} + 0.0019D_{t-4}$$

2) $$TD_t^d = -0.00026(i_tY_t) + 0.0032(r_{S\&T_t}Y_t) + 0.028Y_t - 0.0016(N_tP_t)$$
$$+ 0.87TD_{t-2}^d$$

3) $D1^m_t \equiv D_t - IBD^m_t - Float_t + FD_t$

4) $TD^d_t \equiv S_tD1^m_t - DG_t$

5) $S_tD1^m_t \equiv S_t \times D1^m_t$

6) $i_tY_t \equiv i_t \times Y_t$

The endogenous and exogenous variables are the same as those listed in Model I except for two additional variables;

CD^*_{t-2}, which is exogenous and CD^*_{t-3}, which is a lagged exogenous variables in Model II.

The system of simultaneous equations under Model III including equation (98-7) and corrected for autocorrelation is shown below:

1) $\Delta D^*_t = - 264i^*_t + 225.54d^*_t - 5.54r^*_{F_t} - 0.13\Delta T^*_t + 1.45\Delta CL^*_t - 0.27UR^*_{t-1}$

$+ 0.047T^*_{t-2} - 0.18D^*_{t-3} + 0.026CD^*_{t-3} - 0.098U^*_{t-3} - 0.04\varepsilon_{t-1}$

2) $TD^d_t = - 0.00029(i_tY_t) + 0.0033(r_{S\&T_t}Y_t) + 0.029Y_t - 0.00165(N_tP_t)$

$+ 0.87TD^d_{t-1}.$

3) $D1^m_t \equiv D_t - IBD^m_t - Float_t + FD_t$

4) $TD^d_t \equiv S_tD1^m_t - DG_t$

5) $i_tY_t \equiv i_t \times Y_t$

6) $S_t D1_t^m \equiv S_t \times D1_t^m$

7) $\Delta D_t \equiv D_t - D_{t-1}$

8) $\Delta D_t^* \equiv \Delta D_t + 0.145\Delta D_{t-1}$

9) $i_t^* \equiv i_t + 0.145 i_{t-1}$

10) $D_{t-3}^* \equiv D_{t-3} + 0.145 D_{t-4}$

The variables with superscripts (*) are the transformed variables for the autocorrelation correction where $\hat{\rho} = -0.145$ is the value used for transforming the values of the variables.

The above system is reduced to a system of six equations

1) $$D_t = -264 i_t - 38.28 i_{t-1} + 225.54 d_{t-1}^* - 5.54 r_{F_t}^* - 0.13\Delta T_t^* + 1.45\Delta CL_t^*$$
$$- 0.27 UR_{t-1}^* + 0.047 T_{t-3}^* + 0.026 CD_{t-3}^* - 0.098 U_{t-3}^* - 0.04\varepsilon_{t-1}$$
$$+ 0.855 D_{t-1} + 0.145 D_{t-2} - 0.18 D_{t-3} - 0.026 D_{t-4}$$

2) $$TD_t^d = -0.00029(i_t Y_t) + 0.0033(r_{S\&T_t} Y_t) + 0.029 Y_t - 0.00165(N_t P_t)$$
$$+ 0.87 TD_{t-1}^d$$

3) $D1_t^m \equiv D_t - OBD_t^m - Float_t + FD_t$

4) $TD_t^d \equiv S_t D1_t^m - DG_t$

5) $S_tD1_t^m \equiv S_t \times D1_t^m$

6) $i_tY_t = i_t \times Y_t$

Because of the problems mentioned earlier in this part, the Gauss-Seidel forecasting procedure is not employed for the simulation of Models II and III. The results presented for Model I are for illustrative purposes only in this dissertation, but full simulation of all three models is an important area for future analysis.

CHAPTER V. SUMMARY AND CONCLUSIONS

Legal commercial bank reserve requirements have an important influence upon the size of the money stock and the ability of the central bank to control it. Moreover, the selection of the policy variable by the Federal Reserve System has a significant impact on money supply determination. Therefore, the objectives of this study have been: (1) to present the money supply determination process under a lagged reserve accounting system; (2) to develop the above objective assuming different policy variables.

Under the LRA system, the average legal reserves which a bank must hold during a week are a function of the bank's average deposit liabilities held during the week before last. Legal reserves consist of current deposit balances with a Federal Reserve Bank and vault cash held two weeks earlier.

To accomplish the above objectives, three models are formulated based upon a profit maximizing behavioral assumption for commercial banks. Given their known anticipations of the demand and time deposits and the demand for commercial loans, banks try to maximize the expected level of their profits. The decision variables, investments or earning assets, CD liabilities, and Eurodollar liabilities are optimized in the decision making procedure. Then, based upon these optimum values, banks adjust their short-term (one to two weeks) variables, namely, FFM transaction, Repo (RP) and Reverse (RV) transactions, and borrowings from FRB. The desired level of the deposit supply is derived as a result of the above optimization. The deposit supply variations are linked and largely explained by the lagged values of the variables in the models.

The empirical work consists of estimation and prediction of the deposit supply developed in the above models. Deseasonalized, weekly, time series data, for the period of January, 1970, to July, 1976, are used for the estimates. Ordinary-Least-Squares and 2SLS are used to estimate the equations. When the disturbance terms are significantly serially correlated, GLS approach in the context of both the single and simultaneous equation approach is utilized to correct the resulted estimations.

In Model I, investment or earning assets are taken as the decision variable. Banks adjust their FFM transactions and borrowings from FRB based on the optimum values of their investments. Deposit supply is derived as a function of: the policy variable (UR), other exogenous variables (ΔCL and ΔT), the lagged policy variable, lagged demand and time deposits, the three months Treasury bill rate (i), the FRB's discount window rate (d), the required reserve ratios (δ, τ), the ratios for measuring the ability of the banks to forecast accurately the future values of the variables and to adjust for the forecast errors (n_D,n_T,n_{CL}), and a ratio for slow adjustment of free reserves (n_F).

In Model II, the decision variables are investment, CD liabilities, and EURO borrowings. The deposit supply derived from Model II under LRA in Part III is explained by the lagged values of the CD's outstanding in addition to the other explanatory variables used in Model I. In actuality, and especially within the last twenty years, liability management involving the issuance of CD's and borrowings from EURO market has come to be an important instrument for the bank's portfolio adjustment. Therefore, Model II is able to formulate reality more accurately.

After the mid-sixties, the kind of instructions given by the FOMC for implementing OMO showed a narrower band for controlling the FFR's variations. Therefore, it seems that the FOMC has controlled the FFR exogenously as a short-run instrumental variable to control the money supply path. Reserve aggregates played a dominant role as short-run instrumental variables before the mid-sixties; see Stigum (37). In Models I and II, a measurement of the aggregate banking reserves, namely, UR_t, is taken as an exogenous and short-run instrumental variable. In Model III, the FFR is considered as the short-run instrumental variable used by FOMC, exogenously, to control the money supply. RP and RV transactions are utilized as short-term (one to two weeks) decision variables by banks in Model III. Based on the optimum values of the decision variables, investment, CD, and EURO borrowing, banks adjust their FFM transactions, borrowing from FRB, and RP or RV transactions. The deposit supply derived from Model III is a function of the policy variable (FFR), in contrast to (UR), and other explanatory variables used in Model I.

Empirical results from Models I and II indicate that the variation of the change in the deposit supply is significantly explained by the policy variable UR and the change in the nonbank-public's demand for commercial loans (ΔCL). This result supports the hypothesis that banks accommodate the increase in the demand for commercial loans in the short-run. In other words, the supply of commercial loans is demand determined.

The term ΔCL also plays a significant role in Model III because of the above reasons. The policy variable FFR has no significant power in explaining the change in deposit supply in Model III. The high correlation among the policy variable FFR and other interest rates might be responsible for causing this problem. Results from the models show a significant degree of explanatory power for the lagged deposits D, T, and CD. This supports the idea that under the LRA system, the past activities of the banks play an important role in their current and future decision making processes.

The rate of return on earning assets (i) for the representative banks in all models consists of only the monetary interest rate. As emphasized in the current literature (4, 16, 29), the expectations of the future rates of return play an important role in the bank's decision making process. Moreover, the explicit monetary discount rate (d), as a measure of the opportunity cost of not borrowing from the FRB, is subject to serious criticism (13, 22, 30, 37). In the estimations for ΔD in all models, the explanatory power for i and d is not significant. This suggests that the expected interest rates and the subjective rates for measuring the true opportunity cost of not borrowing from the FRB are absent as the explanatory variables in the estimation. Inclusion of these variables in the decision making process of the bank could be a subject for future study. This, in turn, may solve the problem of multicolinearity of the policy variable FFR and other rate of return observed in Model III.

The issuance of CD's and/or borrowings from EURO market, as an instrument for the bank's portfolio adjustment, has been considered in Models II and III, to explain more accurately the observed facts. As seen in Models II and III under LRA, the current changes in CD and U have no effect on ΔD. The manner in which CD and U are integrated in the models channels the effects of the changes in CD and U upon ΔD through the changes in the bank's reserves. Therefore, CD and U can only affect ΔD after two weeks.

The signs for the coefficients of CD_{t-2}, CD_{t-3}, U_{t-2}, and U_{t-3} in the estimates of ΔD from the empirical work contradict the theoretical results. This suggests that there are additional direct effects by the current and lagged CD's and U's on ΔD which are not undertaken by the models. More complete analyses of the CD and U markets and of the way that CD and U liabilities are integrated in the decision making process by the banks may solve the above problems.

Although the correction for autocorrelation improved the regression results in all estimations, the explanatory powers and the direction of the effects of the variables on ΔD remained the same. The estimate of ΔD within the context of the simultaneous equation approach changed the explanatory power and the sign of the coefficient of the market interest rate (i). These changes, however, did not confirm the theoretical results. The functional form of the demand for demand deposit function is not analyzed in this study. That may be another reason for the above behavior.

On empirical grounds, the following areas for further research and analysis are suggested. First, a comprehensive analysis of the demand deposit demand equation should be undertaken. Second, trends and fluctuations in the financial time series data should be analyzed by using available techniques such as spectral analysis and the Box-Jenkins method. These techniques could optimize the procedure for deseasonalizing the time series data. Third, OLS and 2SLS are utilized for estimating the models. Limited information maximum likelihood technique and three-stage-least-square may be employed to improve estimations. Fourth, the Gauss-Seidel algorithm for a period of one year from July, 1975, to July, 1976, is utilized for simulation of Model I. The entire period of estimation undertaken by this study is not employed for simulation purposes because of the capacity constrained for the available Gauss-Seidel program. The increase of the capacity of the above program to simulate the models for the entire period of this study can be a subject for future study. Fifth, weekly time series data have been used for the period January, 1970, to July, 1976. As data become available, this study can be updated to the current time period.

BIBLIOGRAPHY

1. Aigner, Dennis J. "On Estimation of an Econometric Model of Short-Run Bank Behavior." Journal of Econometrics 1 (Oct. 1973):201-28.

2. Aigner, Dennis J. and Bryan, William R. "A Model of Short-Run Bank Behavior." Quarterly Journal of Economics 85 (Feb. 1971):97-118.

3. Board of Governors of the Federal Reserve System. The Annual Report of the Board of Governors. Washington: Board of Governors, Issues 1914 and 1915.

4. Boltensperger, Ernst and Milde, Hellmuth. "Predictability of Reserve Demand, Information Costs, and Portfolio Behavior of Commercial Banks." Journal of Finance 31 (June, 1976):835-43.

5. Bowerman, Bruce L. and O'Connell, Richard T. Forecasting and Time Series. North Scituate, Massachusetts: Duxbury, 1979.

6. Bryan, William R. and Carleton, Willard T. "Short Run Adjustments of an Individual Bank." Econometrica 35 (April, 1976):321-47.

7. Bryan, William R. and Carleton, Willard T. "Deposit Expansion and Federal Reserve Banking System Interaction: A Micro Unit Simulation." American Economic Review 61 (Dec. 1971):901-15.

8. Burger, Albert. "Lagged Reserve Requirements: Their Effect on Federal Reserve Operations, Money Market Stability, Member Banks, and the Money Supply Process." Unpublished paper for the Federal Reserve Bank of St. Louis.

9. Chandler, Lester V. The Economics of Money and Banking. 6th Ed. New York: Harper and Row, 1973.

10. Christ, C. Econometric Methods and Models. New York: Wiley and Sons, 1966.

11. Coats, Warren L. "Regulation D and the Vault Cash Game." Journal of Finance 28 (June, 1973):601-7.

12. Coats, Warren L. "Lagged Reserve Accounting and the Money Supply Mechanism." Journal of Money Credit and Banking 8 (May, 1975): 167-180.

13. Depamphilis, Donald M. "A Micro Econometric Analysis of the Short-Term Commercial Bank Adjustment Process." Federal Reserve Bank of Boston Research Report #55, 1974.

14. Fair, Ray C. "The Estimation of Simultaneous Equation Models with Lagged Endogenous Variables and First Order Serially Correlated Errors." Econometrica 3 (May, 1970):507-15.

15. Feige, Edgar L. and McGee, Robert. "Money Supply Control and Lagged Reserve Accounting." Journal of Money, Credit, and Banking 4 (Nov. 1977):536-551.

16. Fraser, Donald R. and Rose, Peter S. "Short Run Banks Portfolio Behavior: An Examination of Selected Liquid Assets." Journal of Finance 28 (May, 1973):531-37.

17. Froyen, Richard and Kopecky, Kenneth J. "Reserve Requirements and Money Stock Control with a Flexible Deposit Rate." Unpublished paper, University of North Carolina, 1978-79.

18. Fuller, Wayne, A. Introduction to Statistical Time Series. New York: John Wiley and Sons, 1976.

19. Gilbert, Alton R. "Lagged Reserve Requirements: Implications for Monetary Control and Bank Reserve Management." Review, Federal Reserve Bank of St. Louis 62 (May, 1980):7-20.

20. Havrilesky, T. M., and Boorman, J. Monetary Macroeconomics. Arlington Heights, Illinois: AHM Publishing Corporation, 1976.

21. Henderson, James M. and Quandt, Richard E. Micro Economic Theory. New York: McGraw-Hill, 1958.

22. Hester, Donald D. and Pierce, James L. Bank Management and Portfolio Behavior. New Haven: Yale University Press, 1975.

23. Johnnes, James M. and Rasche, Robert H. "Monetary Control: The Implementation Experience, Retrospective and Prospective." Department of Economics East Lansing, Econometrics Workshop Paper No. 7908, January 1980.

24. Johnston, J. Econometric Models and Methods. New York: McGraw-Hill, 1972.

25. Kopecky, Kenneth J. "The Relationship between Reserve Ratio and the Monetary Aggregates under Reserve and Federal Funds Rate Operating Targets." Staff Economic Studies, Board of Governors of the Federal Reserve, 1978. Summarized in Federal Reserve Bulletin 65 (Dec. 1978):937-38.

26. Laufenberg, Daniel E. "Contemporaneous Versus Lagged Reserve Accounting." Journal of Money, Credit, and Banking 8 (May, 1976):239-245.

27. Laurent, Robert D. "Reserve Requirements: Are They Lagged in the Wrong Direction?" Journal of Money, Credit, and Banking 11 (August, 1979):301-311.

28. Luckett, Dudley G. Money and Banking. New York: McGraw-Hill, 1976.

29. Luckett, Dudley G. "Approaches to Bank Liquidity Management." Economic Review, Federal Reserve Bank of Kansas City 65 (March, 1980): 11-27.

30. Modigliani, Franco, Rasche, Robert, and Cooper, J. Phillip. "Central Bank Policy, the Money Supply and the Short-term Rate of Interest." Journal of Money, Credit, and Banking 2 (May, 1970):166-218.

31. Pierce, David A. "Money Supply Control: Reserves Under Lagged Accounting." Journal of Finance 3 (June, 1976):845-852.

32. Poole, William. "The Making of Monetary Policy: Description and Analysis." Economic Inquiry 4 (June, 1975):253-265.

33. Poole, William. "A Proposal for Reforming Bank Reserve Requirements in the United States." Journal of Money, Credit, and Banking 8 (May, 1976):137-147.

34. Scarfe, Brian L. Cycles, Growth and Inflation. New York: McGraw-Hill, 1977.

35. Starleaf, Dennis R. "The Specification of Money Demand-Supply Models which Involve the Use of Distributed Lags." Journal of Finance 4 (Sept., 1970):743-760.

36. Stephenson, James A. and Farr, H. T. "Seasonal Adjustment of Economic Data by General Linear Statistical Model." Journal of the American Statistical Association 67 (March, 1972):37-45.

37. Stigum, Marcia. The Money Market Myth and Reality. Homewood, Illinois: Dow Jones, Irwin, 1978.

38. Wood, John H. Commercial Bank Loan and Investment Behavior. New York: John Wiley and Sons, 1975.

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APPENDIX I: FEDERAL FUND MARKET

Federal Fund Market under the Aggregate Reserves Instruments

According to the assumptions concerning behavior considered in Chapter III, the banks with positive free reserves have surplus and no borrowing, and the banks with negative free reserves have borrowing and no surplus. In the aggregate, if FR > 0, it means that a larger number of the banks have surplus or the larger banks have surplus. If FR < 0, then a larger number of the banks have borrowing or the larger banks have borrowing. If FR = 0, it means that some banks carry surplus and some borrowings, but in the aggregate, the surplus is equal to the borrowing.

$$FR = S-B^* \tag{1}$$

On the average, as FR increases, it is expected that there are more banks with surplus and fewer banks with borrowing in the system. S is expected to increase and B^* in absolute value is expected to decrease. Moreover, as FR increases, the increase in S is limited to the value of FR and the decrease in the absolute value of B^* is limited to zero. On the other hand, as FR decreases, S would fall to a limited value equal to zero and the absolute value of B^* would rise to the limited value equal to FR. It is known that:

$$\Theta \leq \frac{\partial S}{\partial FR} \leq 1$$

$$-1 \leq \frac{\partial B^*}{\partial FR} \leq 0.$$

In reality, other factors might disturb these relationships. For example, the FFM transactions might rise at the time that FR increases. This would result in an increase of S even greater than the increase in FR or, in other words,

$$\frac{\partial S}{\partial FR} > 1.$$

On the average, the above relationships are valid in the banking system.

Under LRA, given RR_t, the level of FR_t in the banking system is determined by the policy variable UR_t. Any change in UR_t would result in a change in FR_t by the same amount in the same direction. So, under the LRA system:

$$S_t = S(UR_t), \qquad 0 \leq \frac{\partial S_t}{\partial UR_t} = S' \leq 1, \tag{2}$$

$$B_t^* = B^*(UR_t), \qquad -1 \leq \frac{\partial B_t^*}{\partial UR_t} = B^{*'} \leq 0. \tag{3}$$

On the other hand, since UR_t is used as a control variable, FFR would be determined endogenously in the FFM without outside or exogenous constraints. The rate for the cost of the borrowing r, in this case, can be approximated by the FFR, which is the most important component in determining the cost of the borrowing. Therefore, r is defined as FFR.

Defining FFL as the supply of funds in the FFM shows that from the balance sheet of the banking system,

$$FFL_t = S_t - ER_t, \tag{4}$$

where ER_t = excess reserves in the banking system. ER_t depends upon the value of surplus in the banking system and the rate for the cost of borrowing, or FFR or r. Therefore:

$$ER_t = \eta(S,r) \tag{5}$$

where

$$0 \leq \frac{\partial ER_t}{\partial S_t} = \eta_S \leq 1$$

and

$$\frac{\partial ER_t}{\partial r_t} = \eta_r \leq 0.$$

Since

$$S_t = S(UR_t),\ ER_t = E(UR_t, r_t)$$

where

$$0 \leq \frac{\partial ER_t}{\partial UR_t} = ER_{UR} = \eta_S \cdot S' \leq 1.$$

Then,

$$FFL_t = S_t - E(UR_t - r_t) = L(UR_t, r_t) \tag{6}$$

where

$$0 \leq \frac{\partial FFL_t}{\partial UR_t} = L_{UR} = S'(1-\eta_S) \leq 1$$

$$\frac{\partial FFL_t}{\partial r_t} = L_r = -\eta_r \geq 0.$$

Defining FFB as the demand for funds in the FFM, from the balance sheet of the banking system, it follows:

$$FFB_t = B_t^* - B_t, \tag{7}$$

where B_t = borrowed reserves of the banking system from the FRB's discount window. B_t depends upon the value of total borrowing in the banking system, r, and the FRB's discount rate d_t. Thus:

$$B_t = \theta(B_t^*, r_t, d_t) \tag{8}$$

where

$$0 \leq \frac{\partial B_t}{\partial B_t^*} = \theta_B{}^* \leq 1,$$

$$\frac{\partial B_t}{\partial d_t} = \theta_d \leq 0,$$

and

$$\frac{\partial B_t}{\partial r_t} = \theta_r \geq 0.$$

Since

$$B_t^* = B^*(UR_t),$$

then,

$$B_t = \theta\left[B^*(UR_t),\ d_t,\ r_t\right]$$

or

$$B_t = B(UR_t, r_t, d_t)$$

where

$$-1 \leq \frac{\partial B_t}{\partial UR_t} = B_{UR_t} = \theta_{B^*} \cdot B^{*'} \leq 0$$

$$\frac{\partial B_t}{\partial r_t} = B_r = \theta_r \geq 0$$

$$\frac{\partial B_t}{\partial d_t} = B_d = \theta_d \leq 0.$$

Then,

$$FFB_t = B_t^* - B(UR_t, d_t, r_t),$$

or

$$FFB_t = b(UR_t, d_t, r_t), \tag{9}$$

where

$$-1 \leq \frac{\partial FFB_t}{\partial UR_t} = b_{UR} = B^{*'}(1-\theta_{B^*}) \leq 0,$$

$$\frac{\partial FFB_t}{\partial r_t} = b_r = -\theta_r \leq 0,$$

$$\frac{\partial FFB_t}{\partial d_t} = b_d = -\theta_d \geq 0.$$

For the equilibrium in the FFM, it can be stated that:

$$FFL_t = FFB_t \tag{10}$$

or

$$L(UR_t,r_t) = b(UR_t,d_t,r_t).$$

It is then found that FFR is a function of UR_t and d_t;

$$r_t = R(UR_t,d_t) \tag{11}$$

where

$$\frac{\partial r_t}{\partial UR_t} = R_{UR} = \frac{b_{UR}-L_{UR}}{L_r-b_r} < 0,$$

$$\frac{\partial r_t}{\partial d_t} = R_d = \frac{b_d}{L_r-b_r} > 0.$$

The rate of return on the holding surplus r_{s_t} can be expressed as follows:

$$r_{s_t} = r_{E_t}\left(\frac{ER_t}{s_t}\right) + (r_t-c_t)\left(\frac{FFL_t}{s_t}\right), \text{ where} \tag{12}$$

c_t = the transaction cost of transferring funds to the FFM and it is reasonable to assume $r_{E_t} = 0$;[1] thus:

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[1]Decreases in real interest rates due to inflation are not taken into account.

$$r_{s_t} = \left(r_t - c_t\right) \cdot \frac{FFL_t}{s_t} \quad .$$

For the purpose of approximation, since realistically, E_t is quite small

$$\frac{FFL_t}{s_t} \approx 1, \quad \text{so}$$

$$r_{s_t} \approx r_t - c_t \qquad \text{or,} \quad r_t - r_{s_t} \approx c_t. \tag{13}$$

Transaction cost c_t is fairly constant. The relationships above show the difference between the rate of the cost of borrowing and the rate of return on the holding surplus. This difference can be approximated by a constant term. From the theoretical work in Chapter III, the desired value for the anticipated level of the free reserves is:

$$\overline{FR}^*_t = -H\left(\frac{i_t - r_{s_t}}{r_t - r_{s_t}}\right) = -H\left(1 - \frac{i_t - r_t}{r_t - r_{s_t}}\right) \; ;$$

therefore, it can be written:

$$\overline{FR}^*_t = -H\left(1 - \frac{i_t - r_t}{c_t}\right) \qquad \text{or} \qquad \overline{FR}^*_t = f(i_t - r_t), \tag{14}$$

where

$$\frac{\partial \overline{FR}^*_t}{\partial i_t} = - \frac{\partial H\left(\frac{i_t - r_{s_t}}{r_t - r_{s_t}}\right)}{\partial\left(\frac{i_t - r_{s_t}}{r_t - r_{s_t}}\right)} \cdot \frac{1}{r - r_s} \quad .$$

From Chapter III, it is known that the functional form H exhibits the same properties as G^{-1}_{J}. G^{-1}_{J} is the functional form for desired levels of the anticipated free reserves for individual bank J. It can then be stated

$$\frac{\partial H\left(\dfrac{i_t - r_{s_t}}{r_t - r_{s_t}}\right)}{\partial\left(\dfrac{i_t - r_{s_t}}{r_t - r_{s_t}}\right)} > 0.$$

From SOC of profit maximization from Chapter III, it is restated $r - r_s > 0$. Thus,

$$\frac{\partial \overline{FR}^*_t}{\partial i_t} = f_i < 0$$

$$\frac{\partial \overline{FR}^*_t}{\partial r_t} = f_r = - f_i > 0.$$

Or, in other words:

$$\frac{\partial \overline{FR}^*_t}{\partial r_t} = - \frac{\partial H\left(\dfrac{i_t - r_{s_t}}{r_t - r_{s_t}}\right)}{\partial\left(\dfrac{i_t - r_{s_t}}{r_t - r_{s_t}}\right)} \cdot \frac{-\left(i_t - r_{s_t}\right)}{\left(r - r_{s_t}\right)^2} ,$$

since

$$i_t > r_{s_t}$$

then

$$\frac{\partial \overline{FR}^*_t}{\partial r_t} = f_r > 0.$$

Alternatively, the desired level of the anticipated free reserves can be written:

$$\overline{FR}^*_t = \Omega(i_t, r_t), \text{ where } \Omega_i < 0, \text{ and } \Omega_r > 0. \qquad (15)$$

The surplus and the borrowing of the banking system can be assumed to be a function of the free reserves. By similar calculations, the functional form R can alternatively be shown as:

$$r = R^a(FR_t, d_t) \qquad (16)$$

where

$$\frac{\partial r_t}{\partial FR_t} = R^a_{FR} = \frac{b^a_{FR} - L^a_{FR}}{L^a_r - b^a_r} < 0,$$

$$\frac{\partial r_t}{\partial d_t} = R^a_d = \frac{b^a_d}{L^a_r - b^a_r} > 0;$$

the superscript "a" indicates the alternative functional forms.

Federal Fund Market under a Federal Fund Rate Instrument

In this part, the FOMC uses FFR as a short-run control variable so FFR is exogenously constrainted and determined. The cost of borrowing is determined endogenously by short-run market forces. Therefore, the rate cost of borrowing cannot be approximated by the FFR. Both UR_t and FR_t are endogenously determined in the system. Under LRA and with the same behavioral assumptions as seen in the preceding section of this appendix for the banking system as a whole, the following relationship can be stated.

$$S_t = S(FR_t), \qquad 0 \le \frac{\partial S_t}{\partial FR_t} \le 1, \tag{17}$$

$$B_t^* = B^*(FR_t) \qquad -1 \le \frac{\partial B_t^*}{\partial FR_t} \le 0, \tag{18}$$

where

$$S_t = ER_t + FFL_t + RV_t \tag{19}$$

$$B_t^* = B_t + FFB_t + RP_t \tag{20}$$

RP_t = Repurchase agreement outstanding at time t.

RV_t = Reverse repurchase agreement outstanding at time t.

Equations (19) and (20) denote that the surplus S_t can alternatively be allocated to ER_t, FFL_t, and/or RV_t, and the borrowing funds B_t^* can be provided as B_t, FFB_t, and/or RP_t.

It can be stated that:

$ER_t = \eta 1(S_t, r_F, r_R)$ where r_F = FFR

and r_R = overnight interest rate in RP (or RV) market.

The excess reserves in the banking system are a function of total surplus in the system, FFR, and the overnight interest rate on RP (or RV). $S_t = S(FR_t)$; thus,

$$ER_t = ER(FR_t, r_F, r_R), \text{ where} \tag{21}$$

$$0 \le \frac{\partial ER_t}{\partial FR_t} \le 1, \qquad \frac{\partial ER_t}{\partial r_F} \le 0, \qquad \text{and} \qquad \frac{\partial ER_t}{\partial r_R} \le 0.$$

Furthermore, $RV_t = \eta 2(S_t, r_F, r_R)$.

The reverse repurchase agreement in the banking system is a function of total surplus in the system, FFR and its own market interest rate. Then

$$RV_t = RV(FR_t, r_F, r_R) \tag{22}$$

where

$$0 \leq \frac{\partial RV_t}{\partial FR_t} \leq 1, \qquad \frac{\partial RV_t}{\partial r_F} \leq 0, \qquad \text{and} \qquad \frac{\partial RV_t}{\partial r_R} \geq 0.$$

On the other hand, on the borrowing side it can be written

$$B_t = \theta 1(B_t^*, d, r_F, r_R).$$

Borrowing from the FRB's discount window is a function of the total amount of borrowing in the system, the FRB's discount rate, FFR, and RP or RV market interest rate.

$$B^* = B^*(FR_t).$$

Therefore,

$$B_t = B(FR_t, d, r_F, r_R), \tag{23}$$

where

$$1 \leq \frac{\partial B_t}{\partial FR_t} \leq 0, \qquad \frac{\partial B_t}{\partial d} \leq 0, \qquad \frac{\partial B_t}{\partial r_F} \geq 0, \text{ and } \qquad \frac{\partial B_t}{\partial r_R} \geq 0.$$

It can also be written

$$RP_t = \theta 2(B_t^*, r_F, r_R).$$

Hence,

$$RP_t = RP(FR_t, r_F, r_R), \tag{24}$$

where

$$-1 \leq \frac{\partial RP_t}{\partial FR_t} \leq 0, \qquad \frac{\partial RP_t}{\partial r_F} \geq 0, \quad \text{and} \quad \frac{\partial RP_t}{\partial r_R} \leq 0.$$

The supply of funds in the FFM can be written as:

$$FFL_t = S(FR_t)-ER(FR_t,r_F,r_R)-RV(FR_t,r_F,r_R),$$

or

$$FFL_t = L1(FR_t,r_F,r_R), \tag{25}$$

where

$$0 \leq \frac{\partial FFL_t}{\partial FR_t} = \frac{\partial S}{\partial FR_t}\left(1- \frac{\partial ER_t}{\partial S_t} - \frac{\partial RV_t}{\partial S_t}\right) \leq 1,$$

$$\frac{\partial FFL_t}{\partial r_F} = - \frac{\partial ER_t}{\partial r_F} - \frac{\partial RV_t}{\partial r_F} \geq 0,$$

$$\frac{\partial FFL_t}{\partial r_R} = - \frac{\partial ER_t}{\partial r_R} - \frac{\partial RV_t}{\partial r_R} \quad .$$

In actuality, the larger response from reverse repurchase transaction to a change in its own interest rate is expected over the response from excess reserves. Therefore, it follows

$$\frac{\partial FFL_t}{\partial r_R} \leq 0.$$

The demand for funds in the FFM can be stated as:

$$FFB_t = B^*(FR_t)-B(FR_t,d,r_F,r_R)-RP(FR_t,r_F,r_R)$$

or

$$FFB_t = b1(FR_t,r_F,r_R,d) \ , \tag{26}$$

where

$$-1 \leq \frac{\partial FFB_t}{\partial FR_t} = \frac{\partial B^*}{\partial FR_t}\left(1 - \frac{\partial B_t}{\partial B_t^*} - \frac{\partial RP_t}{\partial B_t^*}\right) \leq 0,$$

$$\frac{\partial FFB_t}{\partial r_F} = - \frac{\partial B_t}{\partial r_F} - \frac{\partial RP_t}{\partial r_F} \leq 0,$$

$$\frac{\partial FFB_t}{\partial d} = - \frac{\partial B_t}{\partial d} \geq 0$$

$$\frac{\partial FFB_t}{\partial r_R} = - \frac{\partial B_t}{\partial r_R} - \frac{\partial RP_t}{\partial r_R} \quad .$$

Realistically,

$$\left|\frac{\partial RP}{\partial r_R}\right| > \left|\frac{\partial B_t}{\partial r_R}\right| \; ; \quad \text{thus,} \quad \frac{\partial FFB_t}{\partial r_R} \geq 0.$$

The equilibrium condition in the FFM can be written as:

$$FFL_t = FFB_t, \quad \text{or} \quad L1(FR_t, r_F, r_R) - b1(FR_t, r_F, r_R, d) = 0. \qquad (27)$$

Then,

$$FR_t = FR(r_F, d, r_R), \qquad (28)$$

where

$$\frac{\partial FR_t}{\partial r_F} \leq 0, \qquad \frac{\partial FR_t}{\partial d} \geq 0, \qquad \frac{\partial FR_t}{\partial r_R} \geq 0.$$

Both the rate of cost for borrowing r and the rate of return on holding surplus r_s are endogenously determined in the system and are approximated as

$$r \simeq d \cdot \frac{B_t}{B_t^*} + (r_F - C_1)\,\frac{FFB_t}{B_t^*} + (r_R - C_2)\,\frac{RP_t}{B_t^*} \quad , \tag{29}$$

$$r_s \simeq r_E \cdot \frac{ER_t}{S_t} + (r_F - C_1)\,\frac{FFL_t}{S_t} + (r_R - C_2)\,\frac{RV_t}{S_t} \quad . \tag{30}$$

C_1 and C_2 are transaction costs of transforming funds to the FFM and RP market respectively. From the optimization process for the banking system it is understood that

$$\overline{FR}^* = - H\left(\frac{i - r_s}{r - r_s}\right) = - H\left(1 - \frac{i - r}{r - r_s}\right).$$

By substituting for r and r_s, the results can be shown as

$$\overline{FR}^* = - H_1(i, r_F, r_R, d, FR).$$

Since

$$FR = FR(r_F, d, r_R) \quad ,$$

then,

$$\overline{FR}^* = \Omega 1(i, r_F, r_R, d) \quad . \tag{31}$$

In reality, it is expected that

$$\frac{\partial \overline{FR}^*}{\partial i} \leq 0, \qquad \frac{\partial \overline{FR}^*}{\partial r_F} \geq 0, \qquad \frac{\partial \overline{FR}^*}{\partial r_R} \geq 0, \quad \text{and} \quad \frac{\partial \overline{FR}^*}{\partial d} \geq 0.$$

APPENDIX II: DATA

Time Period under Study

The time period undertaken by this study is from the first week of January, 1970, to the last week of June, 1976. Three-hundred thirty-nine (339) observations of weekly data are used for the empirical work within the above time expand.

The reasons for choosing the above time period are:

1) The models in Chapter III best explain the behavior of banks and Fed (or FOMC) during the decade of the Seventies.
2) Lack of available weekly data on essential variables after June of 1976.

The models in Chapter III formulate the short-run banking decisions which at best can be considered as weekly decisions. Therefore, using weekly data is crucial.

Data Directory

Unless specified, all data are weekly averages, not seasonally adjusted figures, and measured in terms of millions of dollars, with the exception of interest rates and ratios.

Variables:

i Treasury bill interest rate - the rate which 99-day Treasury bills are discounted in the open money market.

d Discount rate - the rate at which the New York FRB will rediscount eligible paper of member banks and advances on promissory notes secured either by such eligible paper or by USGS's.

$r_{S\&T}$ Composite interest rate on time and savings deposits - this series is interpolated from monthly $r_{S\&T}$ series.

r_F Federal funds interest rate - the rate at which excess reserves on deposit with Fed are traded by member banks.

UR Unborrowed reserves - the total reserves of member banks minus borrowings from FRB's discount window.

RR Required reserves.

FR Free reserves.

T Time and savings deposits at member banks - consist of all time and savings deposits at member banks minus all CD's issued by weekly reporting commercial banks in denominations of $100,000 or more.

CD Certificates of deposit (CD) - negotiable time certificates of deposit issued in denominations of $100,000 or more by large, weekly-reporting commercial banks.

TD^d The demand component of the money stock - the demand deposits at all commercial banks other than those due to domestic commercial banks and the United States Government, less cash items in the process of collection and FRB's Float.

DG Government demand deposits at commercial banks - the sum of demand balances of the United States Government at commercial banks.

DG^m Government demand deposit at member banks - the sum of demand balances of the United States Government at member banks.

ODD^m Other demand deposits at member banks - gross demand deposits at member banks minus the sum of demand deposits due to banks and the United States Government demand deposits at member banks.

FD Foreign deposits at FRB's.

Due to^m Due to member banks - demand deposits due to member banks that are reported by commercial banks in the United States.

Due $from^m$ Due from banks at member banks - demand deposits due from commercial banks in the United States that are reported by member banks.

$CIPC^m$ Cash items in the process of collection at member banks - the amounts that member banks have credited to depositors' accounts but have not yet collected from the banks on which the deposited checks were drawn.

Float Float - reserve credit given for checks not yet collected. It represents cash items in the process of collection at FRB's less the sum of deferred availability cash items and all collected funds due other reserve banks which have not yet been remitted.

CL Commercial loans - all commercial and industrial loans lent by large commercial weekly-reporting banks, which account for 90% of all commercial and industrial loans lent by weekly-reporting banks. This item is a Wednesday figure.

Y Gross national product (GNP) - the current dollar value of gross national product, seasonally adjusted quarterly data. Variable Y is redefined as a weekly average data by linear interpolation of quarterly time series data. The method for interpolation is as follows:

Given the values for two subsequent quarters Y_0 and Y_{13}, the linear equation is written as

$$Y = \frac{Y_{13}-Y_0}{13-0} X - \frac{Y_{13}-Y_0}{13-0} (13) + Y_{13}.$$

Plugging the values of X = 1, 2, 3, ..., 13 in the above equation, the subsequent weekly average values for Y (namely Y_1, Y_2, Y_3, ..., Y_{13}) are derived. Similarly for the other quarters for X = 14, ... the values for Y_{14} ... are derived which results in the different equations for different quarters.

P GNP price deflator, base year = 1972, seasonally adjusted, quarterly data. The variable P is redefined as weekly average data by linear interpolation of quarterly time series data. The above method is used for interpolation.

N Population, quarterly data, the variable N is redefined as a weekly average data by linear interpolation of quarterly time series data. The above method is used for interpolation. N is measured in terms of one person.

U Eurodollar borrowings - gross liabilities of banks to their foreign branches by large, weekly-reporting commercial banks.

RR_D Required reserves against net demand deposits.

RR_{T+CD} Required reserves against total time and savings - the required reserves against savings and time deposits and total CD liabilities.

RR_U Required reserves against EURO's.

Derived variables:

NP Reverse of per capita index.

$NP = N \times P$.

IBD^m Intermember bank deposits.

$IBD^m = \text{Due to}^m - \text{Due from}^m$.

D Net demand deposits at member bank where

$$D = DDA^m + DG^m + IBD^m$$

$$DDA^m = GDD^m - DG^m - \text{Due to}^m - CIPC^m\text{, therefore,}$$

$$D = GDD^m - \text{Due from}^m - CPIC^m\text{, where}$$

GDD^m = gross demand deposits at books of member banks.

DDA^m = demand deposits adjusted at member banks.

It is defined:

$$ODD^m = GDD^m - \text{Due to}^m - DG^m\text{; thus,}$$

$$D = ODD^m + DG^m + IBD^m - CIPC^m.$$

$D1^m$ Total member bank deposits by nonbank-public and by the United States Government - the sum of (1) the gross demand deposits at all member banks other than those due to member banks that are reported by commercial banks in the United States, less cash items in process of collection and FRBS' Float; and (2) foreign demand balances at FRB.

$$D1^m = GDD^m - \text{Due to}^m - CIPC^m - \text{Float} + FD\text{, or}$$

$$D1^m = D - IBD^m - \text{Float} + FD\text{, or}$$

$$D1^m = ODD^m + DG^m - CIPC^m - \text{Float} + FD.$$

S Blow up factor - the ratio of total deposits by the nonbank-public and the United States Government in the commercial banks to total deposit of the nonbank-public and the United States Government in the member banks.

$$S = \frac{TD^d + DG}{D1^m} .$$

δ Required reserve ratio against net demand deposit

$$\delta = \frac{RR_D}{D_{t-2}} \quad \text{under LRA.}$$

τ Required reserve ratio against time deposit.

$$\tau = \frac{RR_T}{T_{t-2}} \quad \text{under LRA.}$$

γ Required reserve ratio against CD.

$$\gamma = \frac{RR_{CD}}{CD_{t-2}} \quad \text{under LRA.}$$

ω Required reserve ratio against U.

$$\omega = \frac{RR_U}{U_{t-2}} \quad \text{under LRA.}$$

$\tau 1$ Composite required reserve ratio against T and CD.

$$\tau 1 = \frac{RR_{T+CD}}{T_{t-2} + CD_{t-2}} \quad \text{under LRA.}$$

RL Reserve release

$$RL = - [\Delta\delta \cdot D_{t-3} + \Delta\tau 1 \cdot (T_{t-3} + CD_{t-3}) + \Delta\omega \cdot U_{t-3}].$$

Data Sources

1. Quarterly data on Y and P in terms of current values are obtained from *Survey of Current Business*.
2. Quarterly data on N are obtained from *Economic Report of the President*.
3. Weekly data on U are obtained from the *Federal Reserve Bulletin*.
4. Weekly data on all remainder variables are obtained from the Board of Governors of the Federal Reserve Banks.